Internal Capital Markets and Corporate Refocusing

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This paper develops a theory of organization based on the benefit and costs of internal capital markets. A central assumption is that the transaction cost of raising external funds is greater than the cost of internal funds. The benefit of internal resource allocation is that it gives the firm a real option to avoid external capital markets (and the associated deadweight transaction costs) in more states of the world than single-business firms. The cost is that internal resource flexibility exacerbates an overinvestment agency problem. The optimal focus is determined by trading off the benefit of the option against the cost of overinvestment. In this context, we show how the relative efficiency of integration and separation depends ultimately on assignment of control rights over cash flow. Testable implications are derived for the level of divisional investment, the sensitivity of divisional investment to cash flow, and the diversification discount. Journal of Economic Literature Classification Numbers: D82, G34, L22. © 2002 Elsevier Science (USA)

Diversification is bad for some firm and good for others. This is one conclusion from a growing empirical literature in corporate finance which find that diversification firms trade at a discount relative to comparable single business firms on average, yet roughly one-third of them trade at a premium. While the basic facts are fairly

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1 We are grateful for helpful feedback from Harry DeAngelo, Anjan Thakor, Jan Zabojnik, and anonymous referees. Some work was completed while the first author was a visiting scholar in the Anderson School at UCLA.

2 Rajan et al. (2000) put the number of diversified firm trading at a premium at 39.3 percent in 1990. Similar numbers can be inferred from the mean and standard errors for the diversification discount reported in Lang and Stulz (1994), Berger and Ofek (1995), and Servaes (1996), among others. The event study evidence has the same flavor: Schipper and Thompson (1983), Matsusaka (1993), and
clear, the reason why some firms are able to profit from diversification while many others are not remains a puzzle. 3

This paper attempts to shed some light on the puzzle by developing a model of organization in which diversification can be efficient or inefficient, depending on characteristics of the firm. The focus of the model is on internal capital markets. We consider two unrelated units that can be operated under a single corporate umbrella or as stand-alone firms. Agency problems and asymmetric information cause firms to overinvest and create a deadweight loss from external finance. In this environment, internal capital markets can be beneficial because they allow headquarters to transfer funds between units without incurring the cost of external finance. But internal capital markets also can be costly because they facilitate overinvestment by insulating headquarters from the constraints of costly external finance. Whether internal capital markets on balance are good or bad for shareholders then depends on how the benefits from avoiding costly external finance stack up against the costs of overinvestment. We show that the benefits can be greater or less than the costs, and therefore diversification can add or destroy value, depending on the relation between the firm’s cash flow and its productive opportunities.

Some elements of our model have been suggested in the literature to be important for understanding corporate diversification, but they have not been brought together and systematically studied. One of our purposes here is to isolate the source of the benefits and costs of internal capital markets. In our model, integrated and stand-alone firms differ only in the way control rights are assigned—in the multidivisional firm, headquarters can transfer cash generated in one unit to another by fiat, while with separated firms such transfers require the acquiescence of both units. In all other respects—the severity of agency problems, financing costs, and ability to identify good investments—integrated and separated firms are the same. Therefore, all of the relevant trade-offs can be traced ultimately to differences in control rights. Put differently, the model shows that both benefits and costs of diversification can arise from differences in control rights alone. This is important because a central theme of the theory of the firm following Grossman and Hart (1986) is that the fundamental difference between integration and separation is the assignment of control rights. 4

The model is also intended to provide a framework for empirical research. It nests as a special case the efficient internal capital market hypothesis, which has been subject to much recent investigation. Absent a well-specified empirical model,
empirical researchers so far have been forced to identify implications based on intuition alone. Our model confirms some of those intuitions, but not all of them. Among other things, we find that internal capital market efficiency (at least as we have formulated it) does not necessarily imply that (i) the multidivision firm accesses external markets less often, (ii) investment of the firm’s high productivity units is less sensitive to cash flow than investment of its low quality units, or (iii) diversified firms invest less (or no more) than comparable firms in their low productivity business. The first conjecture has been mentioned in Comment and Jarrell (1995), the second in Shin and Stulz (1998), and the third in Scharfstein (1997) and Rajan et al. (2000). More generally, the model shows that key relations can go in either direction depending on parameter configurations. This suggests that empirical work will need to condition explicitly on the relevant parameters or employ specifications that allow for nonmonotonocities.

The model also generates a number of novel implications regarding the diversification discount, among them the following.

- The value of internal capital markets depends nonmonotonically on the quantity of internal resources. It might be expected that internal capital markets and hence integration become more valuable as the internal funds that fuel them become more abundant. However, this turns out to be true only to a point. The ability to shift resources between divisions is essentially a real option to avoid (deadweight) external financing costs. When enough resources are available to fund all projects the real option loses its value, and the agency cost of overinvestment comes to dominate.

- Internal capital markets are more valuable as the variability of investment opportunities increases. This follows from the observation that internal capital markets provide a real option: an increase in the variance of the underlying opportunity makes the option more valuable.

- A decline in investment opportunities in a firm’s core business makes diversification less attractive if resources are abundant and more attractive if resources are scarce. The intuition is that as the core business declines, cash flow is freed up to invest in risky noncore businesses. If cash flow is scarce to begin with, this increases the value of the real option. If cash flow is already abundant, the overinvestment problem is aggravated.

Finally, we highlight a previously unrecognized cost of internal capital markets that also springs from the assignment of control rights. It turns out that the flexibility available to the multidivision firm headquarters creates a strategic disadvantage in product market competition. A rival firm will not enter a market if it believes the incumbent firm will be investing heavily in the market. But the multidivision firm cannot commit credibly to a large investment because of its investment flexibility. However, the incumbent can deter entry by allocating a large portion of its cash-generating assets to the threatened market and divesting its other division. Because divestiture raises the cost of shifting resources between divisions (essentially
requiring the deadweight costs of external finance to be incurred), it “locks in” the resources of the threatened unit and effectively precommits to a higher level of investment. In simple terms, another cost of an internal capital market is increased vulnerability to new competition in product markets, and refocusing can be an optimal response to such a threat.  

Our approach differs from previous work in several ways that are worth noting. One strand of the literature has emphasized financial reasons for integration. Lewellen (1971) first called attention to the coinsurance effect of diversification—by reducing cash flow risk, the probability of costly bankruptcy may be lower in a diversified firm. Coinsurance thus can make diversification attractive by allowing integrated firms to borrow at lower cost than stand-alone firms. The basic idea has been extended to allow for various costs of coinsurance arising from debt overhang problems (John, 1993), free cash flow problems (Li and Li, 1996), and asset substitution (risk) problems (Boot and Schmeits, 2000).

Financial considerations are also central to a series of papers that link integration to the informativeness of a firm’s security prices. Aron (1988) suggests that integration can make security prices more informative about the performance of the CEO—the noise associated with unit performance cancels when aggregated—allowing the board to provide better incentives to managers (and presumably lowering the cost of capital as well). On the other hand, Aron (1991), Habib et al. (1997), and Nanda and Narayanan (1999) point out that integration may reduce the quality of information about division performance and exacerbate asymmetric information problems related to equity offerings.

In contrast to this line of research, we assume that integrated and separated firms face the same cost of capital and their security prices are equally informative. Thus, the value of integration does not depend on differences in financing costs or monitoring effectiveness. Instead, we show how the different assignment of control rights in integrated and separated firms alone creates benefits and costs of integration through its impact on the internal capital market.

Another strand of the literature focuses on the impact of control rights on divisional rent-seeking, notably Scharfstein and Stein (2000) and Rajan et al. (2000). In this work, competition for funds within a corporation can distort internal investment decisions. We depart from this work by abstracting away from internal influence activities and show how costs of integration can arise in a natural way from the more conventional empire-building problem that is central to much of modern finance.

Finally, our paper provides a new perspective on the “winner-picking” benefit of integration that has been emphasized by Weston (1970), Williamson (1975), and Stein (1997), among others. Stein (1997) is perhaps the best known formalization

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5 An interesting paper by Chemmanur and John (1996) analyzes the refocusing decision from the viewpoint of an entrepreneur/manager who wants to maximize the chance that he retains control of two projects.

6 Other work in this vein includes Meyer et al. (1992), Fulghieri and Hodrick (1998), and Wulf (1998).
of the idea. In his approach, outside investors have less information than managers about project quality so external capital markets do a poor job picking winners and avoiding losers. Internal allocation does better because the CEO has superior information about project quality and the right incentives since his private benefits are assumed to be positively correlated with overall returns. Our model, in contrast, does not assume that internal capital markets lead to more informed decisions. Instead, we suggest they are better able to take advantage of disparities in investment opportunities. Internal capital markets, in our view, have greater flexibility to redistribute funds because they bypass the frictions associated with external financing more often.

Section 1 describes the basic model and main assumptions. Section 2 investigates and compares the investment policy of multidivision and separated firms. Section 3 analyzes the trade-offs between the two forms of organization and the determinants of the diversification discount. The basic model is extended in Section 4 by introducing a potential entrant. Section 5 concludes.

1. THE BASIC MODEL

We study a 3 period model of a firm with assets in place and investment opportunities in two businesses, $i = 1, 2$. The organization decision, whether to continue operation as a multidivision corporation or refocus and split into two firms, is made in period 0. In period 1, the managers decide how much to invest in each business and arrange financing. The managers seek to maximize expected net proceeds plus private benefits that they receive from investment per se. In period 2, the investments generate cash and the shareholders receive a liquidating dividend. The key elements of the model are summarized in Fig. 1.

*Organization and Assets in Place*

The firm begins with assets in place that generate a deterministic quantity of liquid resources, $w$, in period 1. If the firm refocuses in period 0, the assets are divided between the two units. The natural interpretation of this event is that one division is spun off. Also in period 0, any quantity of the assets can be liquidated and the proceeds distributed in the form of dividends.

*Investment*

Let $I_i$ be the amount of investment in project/business $i$. The return to investment is determined by the market demand curve,

$$P_i = \theta_i - (X_i + Y_i),$$

where $P_i$ is the market price, $X_i + Y_i$ is market output, and $\theta_i$ is a shift parameter representing market demand or more generally investment productivity. Market
output is the output of the firm in question, $X_i$, plus the combined output of all other firms, $Y_i$. The marginal cost of output is constant and equal to 1, so that investment is converted into output according to $X_i = I_i$. The revenue (gross cash flow) from investment in industry $i$ is then

$$R_i(I_i; \theta_i, Y_i) = P_i X_i = (\theta_i - Y_i - I_i) I_i.$$  

Except where noted, we assume that $Y_1 = Y_2 = 0$. Our formulation incorporates no complementarities between division 1 and division 2, and therefore it can be thought of as the case of an “unrelated” diversified firm or a pure conglomerate.

To model the notion of stochastic investment opportunities, we assume that the returns from investing are stable in industry 1 and variable in industry 2. The demand parameter in the “stable” industry, $\theta_1$, is deterministic. The demand parameter in the “variable” industry, $\theta_2$, takes one of two values, $\theta_H$ or $\theta_L$, with equal probability ($\theta_H > \theta_L$). The realization of $\theta_2$ becomes known at the start of period 1, that is, after the organization decision has been made, but before investment and financing.$^7$

$^7$ We do not address the issue of how the managers acquire information about divisional investment opportunities. This problem has been studied by Thakor (1990) and Harris and Raviv (1996), among
Managers, not shareholders, choose the level of investment. We introduce an agency problem by assuming that managers derive a private benefit per dollar of investment. Incentive contracts are presumed to be incomplete, so that instead of maximizing shareholder value, managers maximize the sum of shareholder value and their private benefit from investment. The private benefit from investment, and thus the severity of the agency problem, is the same for multidivision and separated firms.

**Financing**

Investment is financed internally with resources generated by existing assets and, if necessary, externally by raising funds from outside investors. The transaction cost associated with external acquisition of resources is per unit. That is, if the firm raises externally, then it suffers a deadweight cost of . The cost is the same for all firms.

Our assumption that external finance entails additional costs compared to internal finance places the model in the company of a large literature that shows how asymmetric information and agency problems cause a divergence between the internal and external cost of funds. What is nonstandard in our approach is the assumption that the deadweight costs are linear in the amount borrowed and do not depend on the form of organization. The main purpose of the assumption is expositional. By keeping the deadweight costs the same for multidivision and separated firms, we are able to isolate the effect of control rights. As we discuss below, the main results would appear to be robust to more general cost structures. We show below how a cost structure exactly like the one we use can arise in two optimal contracting models, the first an adverse selection model in the spirit of Myers and Majluf (1984) and the second a costly state verification model that follows Froot et al. (1993).

### 2. FINANCING AND INVESTMENT POLICIES AND FORM OF ORGANIZATION

**A. Financing/Investment of a Single-Business Firm**

The managers make the firm’s financing and investment decisions. To bring out the main tensions in the model, we begin by studying a single-business firm with others, although not in relation to the integration decision. A general result is that capital rationing and other puzzling budgeting behavior can be consistent with value maximization.

8 Such a benefit might arise because of prestige associated with running a large firm, greater availability of perquisites, and so on. This is a common assumption in the literature, for example, Stulz (1990) and Li and Li (1996). Stein (1997) assumes that managerial benefits are positively related to revenue rather than investment; we suspect that the qualitative implications of our model would be similar under this assumption.

9 Specifically, we assume that managerial compensation is independent of investment and firm value.

10 Seminal papers include Townsend (1979) and Myers and Majluf (1984).
(assets that generate) liquid resources \( w \). The value of the firm is

\[
V(I; \theta, w) = R(I; \theta) - I - (I - w)C\lambda(I - w) + w,
\]

where \( \lambda(e) \) is an indicator variable equal to 1 if the firm raises external funds \( (e > 0) \) and zero otherwise. The manager chooses investment to solve

\[
U^*(\theta, w) = \max_I [V(I; \theta, w) + \alpha I].
\]

Let \( I^* \) denote the solution to problem (1). The manager’s investment choice is either \( I^* = w \) or the solution to

\[
\frac{\partial R(I^*; \theta)}{\partial I} + \alpha = 1 + C\lambda(I^* - w),
\]

which is simply the familiar condition that marginal benefit equals marginal cost (from the manager’s perspective). The marginal benefit is marginal revenue, \( \partial R_i / \partial I_i = \theta_i - 2I_i \), plus the marginal private benefit of investment, \( \alpha \). The marginal cost of investment is 1 when \( I \leq w \) because \( e = 0 \); the marginal cost is \( 1 + C \) when \( I > w \). Therefore, the marginal cost curve is a step function with the step at \( I = w \).

The solution to problem (1) breaks down into three regions depending on the value of \( w \). Figure 2 illustrates the possibilities. The gray line labeled “MB” is \( \frac{\partial R}{\partial I} + \alpha \). The three marginal cost curves correspond to different levels of \( w \). The first, labeled “MC_low” and corresponding to \( w = w_{low} \), intersects the MB curve at investment level \( I' \). In this case, the firm uses external financing equal to \( I' - w_{low} \). It can be seen that for any \( w \leq I' \), the investment level is \( I' \). The

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**FIG. 2.** The investment decision for a single-business firm.
second marginal cost curve, labeled “MC\text{med}” and corresponding to \( w = w_{\text{med}} \), cuts the MB curve at the step. The frm does not use external fnancing and its level of investment is \( w_{\text{med}} \). For the third marginal cost curve, “MC\text{high},” the frm has more than enough internal resources to fnance its investment. When \( w > I'' \), the investment level is \( I'' \).

In order to economize on notation, we focus our attention on a particular parameter confguration. First, \( \theta_1 + \alpha > 1 + C \); this implies that investment in industry 1 is worthwhile even if it requires external fnancing. Second, \( \theta_L + \alpha < 1 \); this implies that in the low state, industry 2 is so bad that it never pays to invest in it. And third, \( \theta_H + \alpha > 1 + C \); this implies that it is prof table to invest in industry 2 in the high state even if external fnancing is necessary. Under these assumptions, the solution to the single frm investment problem (1) for \( \theta \in \{ \theta_1, \theta_H \} \) is

\[
I^* = \begin{cases} 
I'(\theta) & \text{if } w \leq I'(\theta); \\
I''(\theta) & \text{if } I'(\theta) < w \leq I''(\theta); \\
w & \text{if } I''(\theta) < w;
\end{cases}
\]  

(2)

where \( I'(\theta) = \frac{1}{2}(\theta + \alpha - 1 - C) \) and \( I''(\theta) = \frac{1}{2}(\theta + \alpha - 1) \). When \( \theta = \theta_L, I^* = 0 \).

An important property of the solution is that investment depends on internal resources. This is a consequence of the assumption that external fnancing is costly. If \( C = 0 \), investment would be \( I^* = \frac{1}{2}(\theta + \alpha - 1) \), independent of the frm’s internal resources. Also note that investment depends on \( \alpha \) when \( w \) is suff ciently small or suff ciently large, but for intermediate values of \( w \) (where investment is determined entirely by internal resources) the agency problem does not distort investment. Finally, we can def ne the value of the frm from the shareholders’ point of view:

\[
V^*(\theta, w) = U^*(\theta, w) - \alpha I^*.
\]  

(3)

B. Refocusing and Financing/Investment of Separated Firms

If the frm refocuses, then the (anticipated cash flow from) nonliquidated assets must be divided between the two divisions. This will determine the investment behavior in period 1 of the descendent single-business frms as indicated in the previous section. The refocusing decision is made in period 0, before the value of \( \theta_2 \) is realized. This is a key difference between the two forms of organization: in separated frms, liquid assets are assigned to the units before the productivity of the investment opportunities becomes known.

We assume that the assets are divided between the surviving frms by the shareholders not managers.\(^{11}\) The assets/resources allocated to the new frms 1 and 2

\(^{11}\) This seems like the natural assumption, but we also worked through the case where the managers divide the assets, and nothing of substance changes.
are \( w_1 \) and \( w_2 \), where \( w_1 + w_2 \leq w \). To determine \( w_1 \) and \( w_2 \), the shareholders solve the following problem, where \( V^5 \) is the expected value of separated firms:

\[
V^5 = \max_{w_1, w_2} \left\{ V^*(\theta_1, w_1) + \frac{1}{2} V^*(\theta_H, w_2) + \frac{1}{2} V^*(\theta_L, w_2) + w - w_1 - w_2 \right\}. \tag{4}
\]

The \( V^* \) functions are defined in Eq. (3), and the other terms represent cash received from assets liquidated at the time of refocusing. Note that the private benefit associated with \( \alpha \) is not a factor in the shareholders’ decision.

Assets are allocated optimally by assigning them to the unit where they have the highest marginal expected value. The marginal value of resources in a single-business firm with a known \( \theta \) is \( \partial V^*(\theta, w)/\partial w \). When \( w < I'(\theta) \), the marginal value is \( 1 + C \), and when \( w \geq I''(\theta) \), the marginal value is 1. The intuition is that each unit of \( w \) has an intrinsic value of 1, and is worth an additional \( C \) in avoided transaction costs when \( w \leq I'(\theta) \). When \( w \) is between \( I'(\theta) \) and \( I''(\theta) \), \( I^* = w \), and \( V^* = (\theta - w)w \). The marginal value in this range is then \( \partial V^*/\partial w = \theta - 2w \).

The solution to problem (4) is given in Lemma 1 in the Appendix, and Lemma 2 summarizes the induced investment behavior of the refocused firms. The pattern is that if the firm refocuses when liquid resources are very scarce, it allocates everything to industry 1. With scarce resources, a unit of the resource allocated to firm 1 offsets \( 1 + C \) in financing costs while a unit allocated to firm 2 offsets \( 1 + C \) only in the high state. When resources are more abundant, marginal units are allocated to firm 2, and when they are more abundant still, marginal units are divided between the two units. Once there are enough resources to drive their marginal value to 1 in each unit, additional assets are liquidated and immediately paid out to shareholders rather than assigned to one of the separated units.

### C. Financing/Investment of the Multidivision Firm

We now investigate the financing and investment behavior of the manager if the firm remains a multidivision organization with a cash flow in period 1 of \( w \).\(^{14} \)

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12. This is an inequality because the shareholders can liquidate assets and pay out the proceeds instead of bestowing them on a surviving firm. We assume that the assets (and thus cash flow) can be divided arbitrarily between the two units. The firm holds cash and claims on a variety of assets (financial assets, real estate, patents, physical plants, receivables, and so on). When the firm separates, it divides the cash and claims to achieve the desired cash flow. Because of transaction costs, shareholders never want to raise external funds at this stage; they would rather wait until the uncertainty is resolved and see if they can avoid external financing altogether.

13. All lemmas and propositions assume that \( \alpha \leq C/2 \). The model’s qualitative features are the same as long as \( \alpha < C \). The case of \( \alpha \geq C \) turns out to be degenerate as discussed in part A of Section III.

14. Recall that in period 0 shareholders can limit the internal funds that will be available in period 1 by liquidating some of the assets.
manager chooses investment levels \( I_1 \) and \( I_2 \) to solve

\[
U^{**}(\theta_1, \theta_2, w) = \max_{I_1, I_2} \{ R_1(I_1; \theta_1) + R_2(I_2; \theta_2) - I_1 - I_2 - (I_1 + I_2 - w) \\
\times C \lambda (I_1 + I_2 - w) + w + \alpha(I_1 + I_2) \}.
\]

(5)

The double asterisks distinguish the multidivision manager’s objective function from that of the single-business manager defined in problem (1). A key feature of the multidivision firm is that internal funds are not committed to a business until after the investment productivities are revealed. The complete solution to problem (5) is presented in Lemma 3 in the Appendix.

The investment policy derived for the multidivision firm is consistent with several empirical regularities. Investment is sensitive to cash flow (because of the cost of external finance).\(^{15}\) The units are interdependent in the sense that a cash flow problem in one unit spills over to the other unit.\(^{16}\) The interdependence takes the form of winner picking: units with better opportunities are given access to more resources and invest more.\(^{17}\)

Lemma 3 also provides some insight into regressions that look at the sensitivity of investment to cash flow, after controlling for \( q \). In those regressions, the coefficient on cash flow corresponds to \( \partial I / \partial w \) in our model. The model produces the fairly intuitive implications that investment depends positively on total cash flow and the quality of the unit’s investment opportunities. Less intuitively, the model implies a negative relation (when only internal funds are used) or no relation (when external funds are used) between investment and the productivity of the other unit. The model does not imply that investment is less sensitive to cash flow in the high productivity unit than the low productivity unit. Shin and Stulz (1998) conjecture that such behavior would arise in an efficient internal capital market based on the intuition that internal allocation will buffer the most productive division from the uncertainties of divisional cash flow. This does not happen in a model like ours because optimal investment requires equating the marginal product of investment across units. Given that both divisions have equal marginal products, it would never be optimal to apportion a marginal dollar of cash flow only to one unit. The upshot of this is that the evidence in Shin and Stulz (1998) that investment in the best division is just as sensitive to cash flow as investment in the worst division does not weigh against the view that internal capital allocation is efficient.

The model does imply that the sensitivity of investment to cash flow is non-monotonic in the level of internal resources. When resources are scarce, the firm taps external markets for funds and investment does not depend on cash flow. As resources increase, internal funds determine investment and sensitivity rises. When resources become large enough to fund all investments, the sensitivity falls


\(^{16}\) See Lamont (1997) and Shin and Stulz (1998).

\(^{17}\) Evidence showing more investment in high productivity (as measured by \( q \)) divisions appears in Rajan et al. (2000).
to zero again. Thus, our model supports the general point of Kaplan and Zingales (1997) and Almeida (1999) that the sensitivity of investment to cash flow is not a good indicator of resource constraints. Our model could be tested by dividing firms into (say) deciles according to their internal resources and then estimating the sensitivity of investment to cash flow in the usual way for each decile. The model implies low sensitivity in the bottom deciles, higher sensitivity in the middle deciles, and lower sensitivity in the top deciles.

Denote the manager’s investment choices in industries 1 and 2 in the low state as $I_{1L}^{**}$ and $I_{2L}^{**}$, respectively, and in the high state as $I_{1H}^{**}$ and $I_{2H}^{**}$, respectively. The multidivision firm’s expected value, which we define as $V^M$, can be expressed as

$$V^M = \frac{1}{2}U^{**}(\theta_1, \theta_H, w) + \frac{1}{2}U^{**}(\theta_1, \theta_L, w) - \frac{1}{2}\alpha(I_{1H}^{**} + I_{2H}^{**} + I_{1L}^{**} + I_{2L}^{**}) - \alpha.$$ (6)

This is simply the expected value of the manager’s objective function less the manager’s expected private benefit from investment.

D. Comparison of Multidivision and Refocused Firm Investment Policies

Table I summarizes and compares the financing behavior of multidivision and refocused firms. There are seven distinct regions (labeled with Roman numerals) defined by the amount of liquid resources of the parent firm. The financing policy is indicated for each region, state, and form of organization. The three options are

| TABLE I |

| Definition of Liquid Resource Regions and Financing Behavior of Multidivision and Separated Firms in Each Region |

<table>
<thead>
<tr>
<th>Region</th>
<th>Range of $w$</th>
<th>High state</th>
<th>Low state</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Multidivision</td>
<td>Separated</td>
</tr>
<tr>
<td></td>
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<td>f rm</td>
<td>f rm 1</td>
</tr>
<tr>
<td>I</td>
<td>Less than $I'(\theta_1)$</td>
<td>E</td>
<td>E</td>
</tr>
<tr>
<td>II</td>
<td>$I'(\theta_1)$ to $w^0$</td>
<td>E</td>
<td>IC</td>
</tr>
<tr>
<td>III</td>
<td>$w^0$ to $I''(\theta_1)$</td>
<td>E</td>
<td>IC</td>
</tr>
<tr>
<td>IV</td>
<td>$I'(\theta_1)$ to $I'(\theta_H) + I''(\theta_H)$</td>
<td>E</td>
<td>IC</td>
</tr>
<tr>
<td>V</td>
<td>$I'(\theta_1)$ to $I'(\theta_H)$ to $w^0 + I''(\theta_H)$</td>
<td>IC</td>
<td>IC</td>
</tr>
<tr>
<td>VI</td>
<td>$w^0 + I'(\theta_H)$ to $I'(\theta_1) + I''(\theta_H) - \alpha$</td>
<td>IC</td>
<td>IC</td>
</tr>
<tr>
<td>VII</td>
<td>Greater than $I''(\theta_1) + I''(\theta_H) - \alpha$</td>
<td>IU</td>
<td>IU</td>
</tr>
</tbody>
</table>

Note. This table assigns Roman numerals to regions of the liquid resource $w$. In addition, for each region it identifies the investment and financing behavior of the different forms of organization. E means that the firm uses external financing. IC means that the firm uses only internal funds, and its investment is constrained so that all of its internal resources are invested. IU means that the firm uses only internal funds, and its investment is unconstrained so that it has resources beyond what it needs for investment. A zero means that the firm invests nothing.
external financing (E), internal financing with constrained investment (IC), and internal financing with unconstrained investment (IU). If investment is constrained, the firm invests all available liquid resources, while if it is unconstrained, resources remain after all investment needs are met and are paid out to shareholders. It is apparent that the propensity to raise funds externally depends on the form of organization; this is one reason why refocusing can matter.

Here we note some empirical implications of the model concerning the relative investment and financing policies of diversified and separated firms.

1. **External financing.** Comment and Jarrell (1995) suggest that a benefit of internal capital markets is avoidance of external financing, but they find little difference in this respect between diversified and focused firms.\(^{18}\) Our model, however, does not imply that diversified firms always employ less external financing. If we compare situations where the firms turn to outside investors, we see that both diversified and separated firms seek external financing in regions I–IV, while only the separated firms do in region V. The story is essentially the same if we compare the quantity of borrowing. For example, the diversified firm borrows more in region I than the combined separated firms. The underlying economics can be understood by first noting that firms tap external financing when their internal funds are insufficient to push the marginal return to investment below \(1 + C\). When resources are not too abundant, the ability of the multidivision firm to transfer resources can make it less likely that the marginal return in each division will obviate external financing. On the other hand, when resources are abundant enough to obviate external financing in all states of the world for the diversified firm, separated firms might find themselves in a situation where one has ample resources and the other does not have enough, which would drive the latter to seek external financing. Bottom line: the use of internal capital markets, even if they are efficient, does not necessarily imply less utilization of external financing.

2. **Investment.** The model exhibits similar behavior for total investment. The multidivision firm invests less than the separated firms in regions II and III, more in regions IV–VI, and the same in regions I and VII. The cause is the mismatch between resources and investment opportunities in the separated firms. When resources are scarce, separated and multidivision firms invest the same amount. As resources increase, separated firm 1 ends up with too many resources in the high state, causing it to invest more than division 1 of the integrated firm. With even more resources, at some point separated firm 1 has too few resources in the low state, causing it to invest less than division 1 of the integrated firm. Finally, when resources become sufficiently large, investment is again the same. This suggests a nonmonotonic relation between internal resources and relative investment.

3. **Investment in high and low productivity units.** Recent evidence suggests that diversified firms invest more in their weaker divisions than single segment

\(^{18}\) They employ three measures for a firm’s reliance on external financing: cash inflow, cash outflow, and short-term debt.
f rms in the same line of business.\textsuperscript{19} Our model can exhibit such behavior (even when $\alpha = 0$), which suggests that it could be consistent with efficient internal capital allocation. To see how this can happen, focus on the high state, where division 1 is the weak unit and division 2 is the strong unit. Here the multidivision f rm invests less in division 1 than independent f rm 1 does if liquid resources are in regions II–IV, and more if resources are in regions V and VI. Thus, the multidivision f rm tends to make relatively less investment in the weak unit when resources are scarce, and relatively more when resources are abundant. The intuition is that differences in investment between multidivision and separated f rms depend on how the liquid resources are endowed. The multidivision f rm ends up investing relatively more in the weak unit when the f rm in the bad industry ends up with too few resources. It would be optimal to shift some of resources from the good f rm (which is what happens in the multidivision f rm), but the separated f rms do not find it efficient to engage in the transfers. Put simply, in our model when the multidivision f rm invests more in the bad unit than the single segment f rm, it is because the single segment f rm is investing too little.

3. THE VALUE OF DIVERSIFICATION AND REFOCUSBING

The expected value of a multidivision f rm relative to refocusing (the diversification premium) is defined to be

$$\Delta = V^M - V^S.$$  

The multidivision f rm is optimal from the perspective of shareholders when $\Delta > 0$. We begin by identifying the benefits and costs of internal capital markets. Then we investigate how $\Delta$ varies in response to a change in parameters. Formal results are stated in propositions, followed by intuitive explanations why they hold. Proofs are contained in the Appendix.

A. *The Benefits and Costs of Internal Capital Markets*

We first establish a central result that links $\Delta$ to the quantity of liquid resources.

PROPOSITION 1. *The diversification premium ($\Delta$) is a continuous function of internal resources ($w$), with the following properties:*

(a) the relation is nonmonotonic: as internal resources increase from zero it is flat, then increasing, then decreasing, and finally flat;

(b) When internal resources are sufficiently small, the diversification premium is zero; for intermediate amounts, the diversification premium is strictly positive; and when internal resources are sufficiently large, the diversification premium is

\textsuperscript{19} For example, see Rajan et al. (2000) and Scharfstein (1997). The quality of investment opportunities is proxied by the $q$ of single segment f rms in the same industry.
negative if an agency problem is present ($\alpha > 0$), and zero if an agency problem is absent ($\alpha = 0$).

The solid curve in Fig. 3 depicts the proposition graphically. An easy way to understand why the curve has this particular shape is to isolate the benefit side of internal capital markets first and then look at how things change when the cost side is introduced.

The pure benefit of internal capital markets appears when there is no agency problem, $\alpha = 0$. The dashed curve in Fig. 3 plots $\Delta(w)$ in this case. As can be seen, internal capital markets are entirely beneficial and the nonmonotonicity is present when $\alpha = 0$. There is a straightforward intuition. When resources are extremely scarce, $w < I'(\theta_1)$, there is no fuel to drive the internal capital market. Both the multidivision and separated firms turn to outside investors. Therefore, the investment levels and value of the firm do not depend on the form of organization. As resources become more plentiful, the multidivision firm becomes efficient because it can direct liquid resources internally to high-value investments, and thereby avoid costly external financing in more states of the world than separated firms. When resources become even more abundant, eventually the firm has enough internal funds to cover most of the investment in both projects. It does little shifting of resources from one unit to the other, and consequently the value of the real option to shift funds begins to decline. Once resources become sufficiently large, $w \geq I'(\theta_1) + I''(\theta_H)$, all profitable projects can be funded internally, and the option has no value.

Now, what effect does introduction of an agency problem have on the value of internal capital markets (from the perspective of shareholders)? One might
expect overinvestment costs to be the same for both forms of organization—we have assumed that $\alpha$ is equal for managers of diversified and refocused firms. However, it turns out that the agency problem has a neutral effect on the relative value of diversification only when resources are sufficiently scarce; otherwise it cuts the value of the integrated firm more. The main formal result is that $\frac{\partial \Delta}{\partial \alpha} < 0$ when $w > I'(\theta_1)$, and $\frac{\partial \Delta}{\partial \alpha} = 0$ otherwise. It can be seen graphically by comparing the two curves in Fig. 3. The proof is contained in Lemma 4 in the Appendix.

The agency problem causes investment distortions when a firm uses external financing (“E” in Table I) or uses internal funds and has cash flow to spare (“U” in Table I). When investment is financed solely from internal funds with nothing to spare (“internally constrained” or “IC” in Table I), investment is the same for multidivision and separated firms. The agency problem takes a bigger toll on the shareholders of multidivision than separated firms because the multidivision firm is less likely to find its investment internally constrained. In the high state, the multidivision firm’s access to two investment opportunities gives it a higher demand for funds, making external financing more likely, while in the low state, the multidivision firm’s ability to shift funds makes it more probable that it will have extra cash after fully investing.\(^{20}\) Intuitively, refocusing builds a wall between units 1 and 2 that discourages the first unit from outside financing in the high state and prevents it from accessing the cash flow of the second unit in the low state.

We can now summarize the rationale for Proposition 1 in terms of the benefits and costs of internal capital markets. The main benefit is the real option embedded in the ability to fund projects without recourse to external financing. The value of this option in the absence of an agency problem is nonmonotonic and nonnegative. With an agency problem, internal capital markets come with costs because the option is exercised by the managers not the shareholders, and the managers have a taste for investment in addition to share value. The agency problem has a negative impact on shareholder value in general, but the cost is greater for the integrated firm because its resource flexibility lowers managers’ opportunity cost of overinvesting. This effect is large enough to make the multidivision firm inefficient when $w$ is large and the real option has little value.

Proposition 1 prompts several observations.

1. A natural question is, which diversified firms are good candidates for refocusing programs? The formal answer is, firms with $\Delta < 0$. Proposition 1 indicates that $\Delta$ is negative—and therefore separation is valuable—when a firm anticipates high levels of cash flow relative to its investment opportunities. Empirically, this

\(^{20}\) The story is a little more complicated in region IV, where internal funds are large enough so that only the separated firm 2 taps outside capital in the high state while the diversified firm is internally constrained. However, this cost of refocusing is overwhelmed by an overinvestment problem in the multidivision firm in the low state.
might show up as a low Tobin’s $q$, which could help explain why refocusing appears to be more common among firms with low $q$’s. More precisely, the model suggests that refocusing is more likely if (i) $q$ is low holding constant liquid resources, or (ii) liquid resources are high holding constant $q$.

(2) As noted, $\Delta$ falls when $\alpha$ increases. Moreover, Lemma 4 shows that when the agency problem becomes severe enough, $\alpha \geq C$, $\Delta$ is nonpositive for all $w$. This suggests that agency problems in themselves can trigger a desire for refocusing on the part of shareholders; not because restructuring removes the underlying agency problem—in our model it remains constant across forms of organization—but because separation can impede managers from acting on their private preferences. Empirically, we expect to see more refocusing in firms where managerial incentives are not aligned with shareholder incentives, such as when managers hold little equity, or when value maximization calls for the firm to downsize.

(3) It is worth noting that the overinvestment problem cannot be solved completely by forcing managers to disgorge cash with a dividend or stock repurchase, or committing them to pay out future cash flows by leveraging up (Jensen, 1986; Stulz, 1990). Such a policy (which we would model as a reduction in $w$) is effective in region VII. However, in the other regions it makes shareholders worse off because managers will respond to a cut in internal resources by raising more outside funds. The higher financing transaction costs will outweigh any savings from reduced investment.

(4) In our model, investment opportunities are random. One would expect that most of our results would go through in a different model in which investment opportunities are fixed and cash flow is random (as in Froot et al. (1993)). Internal capital markets would still have value because they allow the firm to avoid outside financing in more states of the world. An important difference in such a model, however, is that it might be more effective for individual firms to hedge cash flows instead of merging if the transaction costs of hedging are low enough. The variability in our model, which arises from stochastic investment opportunities, is probably not hedgeable using conventional techniques that employ publicly traded or exchange-listed contracts.

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21 For related evidence, see Lang and Stulz (1994), Servaes (1996), Berger and Ofeík (1999), and Schlingemann et al. (1999).

22 Intuitively, when the managers derive such a large benefit from investment, the freewalls in the separated firms are too weak to have an effect.

23 In this respect, our model differs from Stein (1997), where the agency problem is what makes the internal capital market work. One important difference is that our model considers a costly overinvestment problem, while Stein’s precludes this by assumption. The cost of diversification in Stein’s model is an inability of managers to monitor multiple projects. Zuta (1998) also provides a model where refocusing can address investment distortions caused by agency problems. In her model diversified firms have a more limited set of contracts available than single segment firms, so refocusing can help by allowing higher powered incentive contracts.

24 See Denis et al. (1997), Dial and Murphy (1995), and Berger and Ofeik (1999).
B. Transaction Costs

This section studies the relation between \( \Delta \) and the transaction cost of external finance. The main result is stated in the next proposition and depicted in Fig. 4.

**Proposition 2.** The diversification premium \( (\Delta) \) is nondecreasing in the transaction cost of external finance \( (C) \) except when the amount of internal resources falls in a particular (narrow) range (specifically, when \( w^\circ + I'(\theta_H) - \alpha/2 < w < w^\circ + I'(\theta_H) \), where \( w^\circ = \frac{1}{2}(\theta_1 - 1 - C/2) \)).

The reason why \( \partial \Delta / \partial C \geq 0 \) is because the option to avoid external financing becomes more valuable as the transaction cost of raising outside capital rises. For most regions of \( w \), the derivative is strictly positive. It is zero when resources are so low that even the multidivision firm always has to use external financing, and when resources are so abundant that even the separated firms never raise outside funds.

The only unexpected property is the negative derivative when \( w^\circ + I'(\theta_H) - \alpha/2 < w < w^\circ + I'(\theta_H) \). For these \( w \) (a subset of region \( V \)), a rise in \( C \) affects \( \Delta \) only through the value of separated firm 2 because separated firm 1 and the diversified firm never use outside financing. Two offsetting effects come into play as \( C \) increases: value is reduced on account of the higher transaction costs, and value is increased by a reduction in overinvestment. When the indicated condition is satisfied, the total amount of external financing is low enough that the second effect dominates. This anomalous case never occurs when \( \alpha = 0 \), and is possible for only a relatively small set of \( w \).

(1) Proposition 2 provides some support for the argument that efficiencies in external capital markets caused the refocusing movement that began in the
Among the factors that may have contributed to greater efficiency of external capital markets are the increased sophistication and quantitative skills of company analysts, and the greater reliability and abundance of information about company performances and prospects. While plausible, this explanation for refocusing has yet to receive a systematic empirical treatment. Our model suggests one important testable implication of this view: because a change in $C$ has no effect on $\Delta$ when $w$ is very large or very small, transaction cost-driven refocusing should be most common among firms with “moderate” amounts of internal funds. One empirical approach would be to separate firms into deciles according to the amount of internal funds and test whether those in the bottom and top deciles were more likely to refocus than those in the middle deciles.

(2) The proposition suggests that diversified firms should be more common in environments where it is costly to use external capital markets, for example, in partially closed economies (where most funds must be raised domestically, such as Korea) or economies that do not afford strong legal protection to investors. This might go part of the way toward explaining the emergence of conglomerates and conglomerate-like groups in Korea and Japan earlier in this century, and might predict the growth of such firms in developing and post-communist economies, at least until a secure set of property rights has been established.

C. Stable Industry

We next explore the relation between $\Delta$ and the productivity of the stable industry, $\theta_1$. The main result is contained in the next proposition and displayed in Fig. 5.

**Proposition 3.** The diversification premium ($\Delta$) is (a) nonincreasing in the productivity of the stable industry ($\theta_1$) when internal resources ($w$) are sufficiently low, specifically when $w \leq I''(\theta_1) + \alpha/2$, and (b) nondecreasing in the productivity of the stable industry otherwise.

The logic behind this result is as follows. When $\theta_1$ rises, unit 1 becomes a better investment opportunity, and it claims more of the multidivision firm’s internal funds. As a result, there are fewer resources available to exploit the real option if unit 2 ends up with a good investment opportunity. Therefore, the effect of an increase in $\theta_1$ on the value of the real option is similar to that of a decrease in $w$. Consistent with Proposition 1, a decrease in available internal resources (increase in $\theta_1$) tends to reduce the value of an internal capital market when resources are scarce, and increase the value of internal allocation when resources are abundant. The effect is plain in Fig. 5, which shows a fall in $\theta_1$ as a shift to the left of $\Delta(w)$.

(1) Empirical researchers have long noted that many firms diversify following poor performance, or in anticipation of a decline of their home market (for

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25 Bhidé (1990, page 70) claims: “It is primarily the increasing sophistication of capital markets that has eroded the advantages of the conglomerate form, making the diversified corporation a much less valuable institution than it once may have been.”
example, Gort (1962), Weston and Mansinghka (1971), Ravenscraft and Scherer (1987), and more recently Campa and Kedia (1999)). Weston and Mansinghka (1971) branded this behavior “defensive diversification.” Our model can display defensive diversification if we interpret stable industry 1 as the firm’s home industry. Proposition 3 implies that a fall in $\theta_1$ frees up funds for unit 2 which, in a cash-poor company, makes diversification more attractive. Defensive diversification has been interpreted as the consequence of an agency problem, but in our model it can be optimal behavior for a value-maximizing firm as well, that is, it can emerge even if $\alpha = 0$. Interestingly, the model also displays what might be called “defensive refocusing” when liquid resources are abundant: firms refocus in response to a decline in their home industry.\(^{26}\)

Beyond highlighting the logic of such “defensive” organizational responses, the model gives a clean empirical prediction about how firms respond when the prospects of their home industry declines: the resource-poor firms will diversify and the resource-rich firms will refocus. This holds out the possibility of accounting for post-Cold War restructuring in the defense industry, where some firms such as General Dynamics refocused while others such as Lockheed diversified.

(2) A corollary of Proposition 3 is that $\partial \Delta / \partial Y \geq 0$ if $w \leq I''(\theta_1) + \alpha/2$, and $\partial \Delta / \partial Y \leq 0$ if $w > I''(\theta_1) + \alpha/2$. To see why, suppose that other firms increase

\(^{26}\) The model of Meyer et al. (1992) also exhibits a form of defensive refocusing. In that model, when the prospects of one of the firm’s units declines and layoffs become likely, the opportunity cost of spending time in “influence activities” declines for the unit’s managers. Divestiture of the endangered unit can be an optimal response in order to prevent these managers from wasting time lobbying for scarce corporate resources.
their output from zero to $Y > 0$, perhaps as a result of new entry or increased production by existing firms. The addition of $Y$ changes the revenue function of the firm we are studying from $(\theta_1 - I_1)I_1$ to $(\theta_1 - Y - I_1)I_1$, which is formally equivalent to a decline in the industry 1 intercept from $\theta_1$ to $\theta_1 - Y$.

This corollary suggests that increased competition in the form of greater output by competitors can cause firms with scarce resources to refocus. Although many contend that tough new competition can force a firm to divest its divisions in unrelated industries, so far the only equilibrium theory with this property is that of Fershtman and Kalai (1993), which advances the notion that new entrants make competition more “complex,” leading firms to refocus in order to economize on decision-making costs. The intuition of our model is that competition can reduce the marginal value of investment in the home industry and thus make more funds available for managers to overinvest. Consequently, our model provides another way to think about those arguments that attribute the refocusing of the 1980s to increased product market competition due to globalization and deregulation.27

**D. Variable Industry**

Proposition 4 characterizes the relation between $\Delta$ and $\Delta_H$, and Fig. 6 gives a graphical depiction.

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27 For example, a story in *Mergers and Acquisitions* (Waite, 1990, page 82) states that during the 1980s, “[v]irtually every sector of the economy underwent some form of restructuring. The driving force behind the wave of mergers and acquisitions over the past 10 years was the presence of increased competition. This was especially true in the U.S. manufacturing sector, in which intense pressure from international firms forced domestic producers to reflect on their capabilities and their shortcomings.”
**Proposition 4.** The diversification premium \( \Delta \) is nondecreasing in the high-state productivity of the variable industry \( \Delta_H \), and strictly increasing when \( I'(\theta_1) + I'(\theta_H) < w < I''(\theta_1) + I''(\theta_H) - \alpha \).

An increase of \( \theta_H \) corresponds to (i) a rise in the expected demand in industry 2 holding constant the variance, and/or (ii) a rise in the variance holding constant the mean. Proposition 4 could be stated more plainly in the following way: the relative value of internal capital markets rises when either the expected value or variance of demand in the variable industry rises. The basic intuition here follows directly from options theory: a rise in the mean or variance of the underlying opportunity increases the value of the option.

1. Proposition 4 suggests that if a multidivision firm were to choose the characteristics of industry 2, it would prefer an industry with a high variance, other things equal. One of the better documented facts about corporate diversification is that firms tend to diversify into growing industries (Gort, 1962; Weston and Mansinghka, 1971; Ravenscraft and Scherer, 1987). Such behavior would be in the spirit of the model if growing industries are riskier than mature industries, as argued by Gort (1962), or if growing industries have higher expected returns.

2. Some business historians have attributed the rise of the multidivision corporation to innovations in internal organization that made it possible to manage units in more than one industry (for example, see the work of Chandler (1977, 1990).) Our model suggests that the growth of such organizations might be linked as well to more macro factors such as rapidly changing technologies and markets. Increased economic variability might have been the precondition for these organizational innovations to add value.

**E. External Financing Costs in an Optimal Contracting Framework**

Our model assumes that the deadweight cost of external finance displays constant returns to scale (in the amount borrowed) and does not depend on the form of organization. The main reason for this assumption is to keep the costs of external finance the same for multidivision and separated firms. This allows us to isolate the effect of control rights on differences in efficiency between the two forms of organization. Here, in order to suggest that the assumptions are not beyond the pale, we sketch two optimal contracting approaches that would give rise to financing costs consistent with our specification. The first is based on adverse selection and the second on costly state verification.

1. **Adverse selection.** This model draws upon a long tradition of models in corporate finance such as Myers and Majluf (1984). Consider a situation in which managerial quality is unobservable and managers cannot credibly signal their quality to outsiders. A “good” type manager has the objective function given in Eq. (1). A “bad” type manager on the other hand lacks the ability or incentive to produce output and is solely concerned with expropriating corporate resources for personal gain. With probability \( q \), a bad manager succeeds in dissipating all resources
while with probability $1 - q$ he is detected and the resources are preserved. The proportion of good and bad managers in the population is $r$ and $1 - r$, respectively.

The return to investors must compensate for the expected default of firms with bad managers who are not detected. If a debt contract is used and the firm borrows an amount $e = I - w$, then the face value of debt, $D$, must be such that $e = rD + (1 - r)(1 - q)D$. We can restate this as

$$D = e/(1 - q + rq) = e + e\left(\frac{q - rq}{1 - q + rq}\right).$$

Hence, firms seeking to raise funds through external financing face a deadweight cost on account of adverse selection that is linear in the amount raised and does not depend on organizational form. Specifically, $C = (q - rq)/(1 - q + rq)$.28

2. Costly state verification. This model is a variant of the costly state verification model of Townsend (1979).29 Suppose that in addition to the assets discussed above, the multidivision firm has some fixed assets that produce a random output worth either 0 or $x_0$, with probability $q$ and $1 - q$, respectively. If the multidivision firm (denoted 0) refocuses, the two surviving firms, 1 and 2, each receive a claim on a fraction of these assets. Firm $j = 0, 1, 2$ then owns assets that produce 0 or $x_j$ with probability $q$ or $1 - q$ (where $x_1 + x_2 = x_0$). Further, suppose that these are the only assets that are contractible, in the sense that their value can be verified in court, so that any loans must be collateralized against these assets alone. One way to think of this is that the new investment ($I$) is placed in intangible assets like R&D that have no liquidation value.

The value of the collateralizable assets is observable to company insiders at no cost, but the lender, say a bank, must pay $k$ to determine the cash flow by inspection. If the lender inspects, the firm suffers a deadweight cost of $m$, which could include time spent meeting with lenders or time spent in bankruptcy proceedings or loss of reputation.

The optimal contract is that the firm borrows $e$, agrees to repay a (state invariant) amount $D$, and the lender commits to inspect with probability $p$ if the borrower defaults (which happens with probability $q$). In order to deter default, the inspection probability must satisfy $pm = D$. Because the amount to be repaid must earn the

28 A variant of the model allows for costly investigation (auditing) by a lender prior to granting the loan (see Mookherjee and Png (1989).) Within the same structure as above, let $k$ be the cost of inspecting a potential borrower before granting a loan and let $m$ be the cost imposed on a borrower who is discovered to be the bad type (for example, the borrower could be charged with falsifying information.) A mixed equilibrium will exist in which the financial intermediary sets the probability of inspection at $p = e/m$ which deters the bad borrower from applying (observe that this implies $e \leq m$, that is, $m$ is the borrowing capacity.) A good borrower must cover the deadweight cost of inspections, so that $D = e + kp = e + ke/m$. Then $C = k/m$ and the deadweight cost of external finance displays constant returns to scale and no dependence on the form of organization.

29 See also Diamond (1984). This example was inspired by a similar model in Froot et al. (1993).
lender zero profit in equilibrium, $D$ must equal the amount borrowed, $e$, plus the expected deadweight cost, $q_k p$. If we use the fact that $p = D/m$, then $D = e + q_k D/m$. When solved for $D$, this gives

$$D = e + e \left( \frac{q_k}{1 - q_k} \right).$$

Here again the deadweight cost of external finance displays constant returns to scale and does not depend on the form of organization. In this case, $C = q_k/(1 - q_k)$.

4. ANOTHER COST OF INTERNAL CAPITAL MARKETS: COMMITMENT

So far we have developed a theory of organization based on a trade-off between the benefit of flexibility and the cost of overinvestment. In this section, we explore a different cost of internal capital markets—the difficulty of commitment that stems from the flexibility of internal resource allocation. To highlight the point, we assume that $\alpha = 0$ and consider an extension of the model in which the firm can react to the threat of entry into its stable market.

Suppose there is a potential entrant to industry 1. The entrant can pay a fixed cost of $Z \geq 0$ to set up a plant in industry 1. The plant has a capacity of $Y > 0$, and produces at zero marginal cost once built. If the firm enters, it takes the price as given (behaves as a Stackelberg follower) and supplies $Y$ units of output to the market; that is, sales by “other firms” in industry 1 rise by $Y$ units. The entry decision is made after the incumbent chooses its form of organization, but before the uncertainty about $\theta_2$ is resolved, call it period 1/2.

The entrant’s profit, $V^E$, depends on the investment of the incumbent firm

$$V^E(\theta_1 - Y, Z, E[I_1]) \equiv (\theta_1 - Y - E[I_1])Y - Z,$$

where $E[I_1] = \frac{1}{2} I^{*\text{H}}(\text{enters}) + \frac{1}{2} I^{*\text{L}}(\text{enters})$ if the incumbent is a multidivision firm (Lemma 3 with $\theta_1$ replaced by $\theta_1 - Y$), and $E[I_1] = I^*(\text{enters})$ if the incumbent is a single-business firm (equation (2) with $\theta = \theta_1 - Y$ and $w = w_1$). Here “enters” indicates that the investment levels are chosen conditional on entry having occurred.

Entry occurs if $V^E > 0$. If $E[I_1]$ is large enough, the challenger chooses not to enter. The critical level of expected investment that deters entry, $I^d$, is the solution to $V^E(\theta_1 - Y, Z, I^d) = 0$, or $I^d = \theta_1 - Y - Z/Y$.

The multidivision firm is particularly vulnerable to the threat of entry. This is because of its investment flexibility; it can move its liquid resources into industry 2 instead of industry 1. In contrast, a single-business firm cannot reallocate its resources without using costly external markets; its resources are “locked in” to industry 1. As an example, suppose the firm is in region III after entry. If the firm separates prior to entry, all of the resources are allocated to industry 1, and
consequently firm 1 invests \( w \) in both states. If the firm is diversified, it invests \( w \) in industry 1 only in the low state; in the high state it prefers to send resources to industry 2 and invests only \( I'(\theta_1 - Y) < w \) in industry 1. Intuitively, the lack of flexibility in the refocused firm reduces the opportunity cost of investing in industry 1 and “commits” it to a higher level of investment.

What this suggests is that the firm may be able to use its form of organization as a strategic variable. In particular, refocusing/divestiture can function as a commitment device: unit 1 is allocated enough resources to deter entry and unit 2 is divested. It is apparent that there are constraints on the ability of the firm to deter entry. Because firm 1 will choose to invest at most \( I''(\theta_1 - Y) \) if entry occurs, the sub-game perfection requirement limits the possibility of entry deterrence to cases in which \( I'' \leq I'(\theta_1 - Y) \). Also, because the level of investment in firm 1 after entry is always at least \( I'(\theta_1 - Y) \), entry never takes place and there is no benefit from strategic refocusing when \( I'' \leq I'(\theta_1 - Y) \). When \( I'(\theta_1 - Y) < I'' \leq I'(\theta_1 - Y) \), however, entry deterrence may be possible and can be an optimal strategy. It should be noted here that without external financing being costly, such strategic refocusing would not be feasible. If \( C = 0 \) then \( I''(\theta_1 - Y) = I'(\theta_1 - Y) \), and the entrant’s decision to enter would not be affected by the organizational form or the amount of liquid assets in firm 1.

**PROPOSITION 5.** It can be optimal to refocus (divest) to deter entry for some parameter values, specifically, when \( I'(\theta_1 - Y) < I'' \leq I'(\theta_1 - Y) \).

(1) In the discussion following Proposition 3, we noted that entry can make it worthwhile for a cash-rich firm to refocus by shifting down its demand curve. Proposition 5 identifies a second avenue through which increased competition might lead a firm to pursue a refocusing strategy. A diversified firm might want to divest a unit in order to commit investment to a threatened industry and thereby deter entry.

(2) A similar logic provides a link between refocusing and potential exit. To see this, suppose that the multidivision firm is engaged in a line of business that undergoes a permanent decline in demand. Exit by some existing firms will be necessary to attain a new equilibrium. One could model exit by allowing firms the option to cease investing in the industry. Exit occurs when a firm decides not to renew its investment. In such a situation, refocusing may be an optimal way to commit to staying in the industry, and by extension to deter other firms from remaining.

(3) More generally, the model points out that there is a trade-off between flexibility and commitment in a strategic environment. Spencer and Brander (1992)
provide an extensive characterization of the tradeoff between flexibility and commitment in a more abstract setting. Our model tries to make concrete the nature of the benefit from flexibility—the ability to avoid costly external finance—and shows how the choice of organization/divestiture can act as a commitment device in this case.31

(4) As we mentioned, it is impossible for the incumbent firm to deter entry if $I^d > I''(\theta_1 - Y)$. An interesting twist is that it may be advantageous for the firm to have an agency problem in this situation. Recall that the sort of agency problem studied above leads to overinvestment. As $\alpha$ rises, the firm can credibly commit to ever larger investment levels. An agency problem could have strategic value when there is a threat of entry. In fact, it could even be in the interest of the shareholders to create or permit an agency problem, for example, by tying compensation more to investment and less to shareholder value.

5. CONCLUSION

This paper develops a model of the multidivision firm to capture and explore some of the key intuitions about how the relative efficiencies of internal and external capital markets affect the form of organization. One of our purposes is to make precise the idea that diversified firms can be efficient because their internal capital markets allow them to allocate resources better than external markets. In our model, this advantage does not arise because multidivision firms have lower borrowing costs or are better at identifying good investments, but because their internal flexibility allows them to avoid costly external financing in more states of the world than separated firms. That is, the multidivision form of organization gives the firm a real option concerning the financing of its investment opportunities.32 The problem with internal capital markets is that the real option is exercised at the discretion of managers, and this will not always be done in the shareholders’ interest. Because managers care about the value of the firm as well as the private benefit of investment, costly external financing puts a brake on their proclivity to overinvest. Internal capital markets reduce the need for external financing and permit managers to indulge their taste for investment.

The model generates a number of testable implications regarding investment and financing behavior and the diversification discount. Some of the conjectures that others have made based on intuition are confirmed, but others are not supported. Notably, we find that efficient internal capital markets do not necessarily imply that (i) the multidivision firm accesses external markets less often than single segment firms, (ii) investment of the firm’s high productivity units is less sensitive to cash flow than investment of its low quality units, or (iii) diversified firms invest less

31 In a different context, Lewis (1983) also argues that divestiture can act as a commitment device.
32 In this sense, the model is related to the literature on real options, for example, Dixit and Pindyck (1994, 1995).
(or no more) than comparable firms in their low productivity industry. In fact, we find that these relations (and others that have been studied in the empirical literature) can go either way depending on model parameters. This suggests that tests of internal capital allocation theories will need to condition on the relevant parameters (particularly the amount of internal resources) or employ specifications that allow for nonmonotonicities.

Some fairly novel implications about the diversification discount also emerge from our basic model. (i) Contrary to what might be expected, the relation between the value of diversification and the availability of liquid resources is nonmonotonic. (ii) A decline in the prospects of a firm’s home industry can make it profitable for the firm to diversify, as suggested by the literature on “defensive diversification”; but in some cases, poor performance in the home industry can make refocusing optimal. (iii) Because the real option provided by an internal capital market becomes more valuable as the variability of the underlying opportunity increases, diversification becomes more valuable as the investment prospects of the firm’s businesses become more variable.

In an extension of the model, we consider the threat of entry into the firm’s home industry. An interesting result is that a multidivision firm can use refocusing as a way to deter entry. Once division 2 has been divested, the opportunity cost of investing firm 1 resources in industry 1 falls. Divestiture can “lock in” resources in the threatened unit, and commit it to a higher (entry-deterring) level of investment. This identifies another potential cost of internal capital markets: vulnerability to entry threats.

A central theoretical idea underlying the theory of the firm starting with Grossman and Hart (1986) is that the relative efficiency of integration and separation ultimately depends on how control rights are assigned. Our paper is consistent with this understanding of the economics of organization in that the relevant trade-offs between integration and refocusing stem entirely from control rights. Our model attempts to put some flesh on the theory by focusing on a particular control right—the right to direct liquid resources to different investment opportunities—and demonstrating how this right in itself can make internal capital markets efficient or inefficient. The model also can be understood in terms of Coase’s (1937) argument that firms exist to take advantage of transactions that are less costly to make internally than across markets. The model focuses on financial/investment transactions and explores how the transaction costs are determined by the internal and external costs of funds for a firm and managerial agency problems. It thus complements an emerging literature that explores other transaction benefits and costs of internal capital markets, for example, Gertner et al. (1994), Stein (1997), Scharfstein and Stein (2000), Rajan et al. (2000), and Wulf (1998).

Finally, although our model focuses on internal allocation of financial resources, we expect that the arguments are applicable to any resource the firm has that (i) can be allocated across different projects, and (ii) incurs a transaction cost when acquired from external markets. For example, another interpretation of the liquid resource \( w \) would be as a quantity of skilled labor. A defense firm may have a pool of engineers with expertise in satellite technology who can be assigned to
work on projects for the Pentagon or projects for commercial customers. Additional engineers could be hired externally, but only by incurring a transaction cost due to time spent searching, costs of training, adverse selection problems in the labor market, and so on. Analogs of the propositions should go through in this case, which suggests that our results are not entirely dependent on the interpretation of the resource as financial capital.

APPENDIX

**Lemma 1.** The solution to the refocusing/divestiture asset allocation problem (4) is

\[
(w^1, w^2) = \begin{cases} 
(w, 0) & \text{if } w \leq w^\circ; \\
(w^\circ, w - w^\circ) & \text{if } w^\circ < w \leq w^\circ + I'(\theta_H); \\
(w^1, w - \bar{w}^1) & \text{if } w^\circ + I'(\theta_H) < w \leq I''(\theta_1) + I''(\theta_H) - \alpha; \\
(I''(\theta_1) - \frac{1}{2}\alpha, I''(\theta_H) - \frac{1}{2}\alpha) & \text{if } I''(\theta_1) + I''(\theta_H) - \alpha < w;
\end{cases}
\]

where \( w^\circ = \frac{1}{2}(\theta_1 - 1 - C/2), \bar{w}^1 = \frac{1}{2}w + \frac{1}{6}(2\theta_1 - \theta_H - 1), I'(\theta) = \frac{1}{2}(\theta + \alpha - 1 - C), \) and \( I''(\theta) = \frac{1}{2}(\theta + \alpha - 1). \)

**Proof.** The marginal value of resources is deterministic for firm 1. The marginal value of the first \( I'(\theta_1) \) units allocated to firm 1 is \( 1 + C \). The next units have a marginal value of \( \theta_1 - 2w_1 \) until \( w_1 = I''(\theta_1) \). Additional resources have a marginal value of 1.

The marginal value of resources allocated to firm 2 is stochastic. In the low state, firm 2 invests nothing so \( \partial V^\ast(\theta_L, w_2)/\partial w_2 = 1 \) for all \( w_2 \). In the high state, the marginal value of the first \( I'(\theta_H) \) units allocated to firm 2 is \( 1 + C \), giving an expected marginal value of \( 1 + C/2 \) for \( w_2 \leq I'(\theta_H) \). Similarly, the marginal value of units \( I''(\theta_H) \) through \( I''(\theta_H) \) is \( \theta_H - 2w_2 \) in the high state, for an expected marginal value of \( \frac{1}{2}(\theta_H - 2w_2 + 1) \). Finally, the expected marginal value of resources in excess of \( I''(\theta_H) \) is 1 in both states.

The refocusing asset allocation algorithm is then as follows. Begin by allocating resources to firm 1 because the first unit there is worth \( 1 + C \) compared to \( 1 + C/2 \) in firm 2. Continue allocating to firm 1 until the marginal value of resources there falls to \( 1 + C/2 \). This occurs at resource level \( w^\circ \), which solves the problem \( \theta_1 - 2w^\circ = 1 + C/2 \). If resources remain, allocate them to firm 2 until their expected marginal value falls below \( 1 + C/2 \), that is, until \( w_2 = I'(\theta_H) \). Beyond this point until the marginal value equals 1, resources are divided between the firms so that the expected marginal value remains equal between them. If \( \bar{w}_1 \) and \( \bar{w}_2 \) are the asset allocations in this range, they must solve the problem \( \theta_1 - 2\bar{w}_1 = \frac{1}{2}(\theta_H - 2\bar{w}_2 + 1) \), subject to \( \bar{w}_1 + \bar{w}_2 = w \). When \( w \geq I'(\theta_1) + I'(\theta_H) - \alpha \), additional assets given to the firms will be invested by the managers even though the
marginal value of investment is less than 1 from the viewpoint of the shareholders. Therefore, when resources become this abundant, \(w_1 = I''(\theta_1) - \alpha/2, w_2 = I''(\theta_H) - \alpha/2\) and the rest is paid out. ■

**Lemma 2.** Let \(I^*_1\) be the equilibrium investment of firm 1, and \(I^*_2H\) and \(I^*_2L\) be the equilibrium investment of firm 2 in the high and low states, respectively. Given an optimal allocation of assets as indicated in Lemma 1,

\[
(I^*_1, I^*_2H) = \begin{cases} 
(I'(\theta_1), I'(\theta_H)) & \text{if } w \leq I'(\theta_1); \\
(w, I'(\theta_H)) & \text{if } I'(\theta_1) < w \leq w^o; \\
(w^o, I'(\theta_H)) & \text{if } w^o < w \leq w^o + I'(\theta_H); \\
(w_1, w - w_1) & \text{if } w^o + I'(\theta_H) < w \leq I''(\theta_1) + I''(\theta_H) - \alpha; \\
(I''(\theta_1) - \frac{1}{2} \alpha, I''(\theta_H) - \frac{1}{2} \alpha) & \text{if } I''(\theta_1) + I''(\theta_H) - \alpha < w; 
\end{cases}
\]

and \(I^*_2L = 0\); where \(I', I'', w^o, \text{ and } w_1\) are defined in Lemma 1.

**Proof.** The result follows immediately from Lemma 1 and Eq. (2). ■

**Lemma 3.** The solution to the multidivision firm investment problem (5) is

\[
(I^{**}_{1H}, I^{**}_{2H}, I^{**}_{1L}) = \begin{cases} 
(I'(\theta_1), I'(\theta_H), I'(\theta_1)) & \text{if } w \leq I'(\theta_1); \\
(I'(\theta_1), I'(\theta_H), w) & \text{if } I'(\theta_1) < w \leq I''(\theta_1); \\
(I'(\theta_1), I'(\theta_H), I''(\theta_1)) & \text{if } I''(\theta_1) < w \leq I'(\theta_1) + I'(\theta_H); \\
(I''(\theta_1) - \frac{1}{2} \alpha, I''(\theta_H)) & \text{if } I'(\theta_1) + I'(\theta_H) < w \leq I''(\theta_1) + I''(\theta_H); \\
-\frac{1}{2} \alpha, I''(\theta_1)) & \text{if } I''(\theta_1) + I''(\theta_H) - \alpha \leq w; 
\end{cases}
\]

and \(I^{**}_{2L} = 0\); where \(\bar{I}_1 = \frac{1}{2} w + \frac{1}{4} (\theta_1 - \theta_H)\), and \(I'\) and \(I''\) are defined in Lemma 1.\(^{33}\)

**Proof.** The solution to problem (5) must satisfy the first order condition \(\frac{\partial K_1}{\partial I_1} = \frac{\partial K_2}{\partial I_2}\), with the additional requirement that these derivatives are equal to zero except when \(I_1 + I_2 = w\). The optimal investment levels depend on whether the high or low value of \(\theta_2\) is realized.

Consider first the low state. By the assumption that \(\theta_L + \alpha < 1\), there is no investment in industry 2; \(I^{**}_{2L} = 0\). Then investment in industry 1 is given by Eq. (2) with \(\theta = \theta_1\).

Next consider the high state. If internal resources are insufficient to drive the marginal revenue to \(1 + C - \alpha\) in both industries, the firm turns to external

\(^{33}\)Note that \(I''(\theta_1)\) is less than (greater than) \(I'(\theta_1) + I'(\theta_H)\) as \(\theta_H + \alpha\) is greater than (less than) \(1 + 2C\). To focus on a particular case, we have assumed that \(\theta_H + \alpha > 1 + 2C\).
f nancing, and investment in industries 1 and 2 is \( I'(\theta_1) \) and \( I'(\theta_H) \), respectively. If internal resources are suf cient to drive the marginal revenue of investment in both industries below \( 1 + C - \alpha \), then the frm does not use external fnancing. Internal resources are divided between the two units so as to equalize marginal revenue. These investment levels, \( I_1 \) and \( I_2 \), solve \( \theta_1 - 2I_1 = \theta_H - 2I_2 \) subject to \( I_1 + I_2 = w \). If there are enough resources to push the marginal revenue in both industries to \( 1 \), then the investment levels are \( I'(\theta_1) = \alpha/2 \) and \( I'(\theta_H) = \alpha/2 \). It is straightforward to show, in addition, that in period 0 shareholders will liquidate assets in excess of \( w = I''(\theta_1) + I''(\theta_H) - \alpha \).

**Lemma 4.** If \( \alpha < C \), then (i) \( \partial \Delta / \partial \alpha < 0 \) when \( w > I'(\theta_1) \), and (ii) \( \partial \Delta / \partial \alpha = 0 \) otherwise. If \( \alpha \geq C \), then \( \Delta \leq 0 \) for all \( w \).

**Proof.** Suppose that \( \alpha \leq C/2 \). Obviously, a change in \( \alpha \) does not alter \( \Delta \) in region I. In regions II–VII, by the envelope theorem,

\[
\frac{\partial \Delta}{\partial \alpha} = \alpha \left( \frac{\partial E[I^*]}{\partial w} - \frac{\partial E[I^{**}]}{\partial w} \right).
\]

Simple algebra in conjunction with Lemmas 2 and 3 show that \( \partial \Delta / \partial \alpha = -\alpha/4 \) in regions II, III, and V, \( \partial \Delta / \partial \alpha = -\alpha/2 \) in region IV, and \( \partial \Delta / \partial \alpha = -\alpha/4 \) in regions VI and VII. An analogous proof for the case \( \alpha \in (C/2, C) \) is omitted.

Now consider the case \( \alpha \geq C \). For the separated frms, note that shareholders begin by allocating resources to the stable frm, but they never give it more than \( w_1 = I'(\theta_1) \) because the marginal value of investment at \( I'(\theta_1) = 1 + C - \alpha < 1 \). Beyond this, resources are allocated to frm 2 until \( w_2 = I'(\theta_H) \) by the same argument. Otherwise, assets are liquidated. Then frm 1 invests \( I'(\theta_1) \) in both states and frm 2 invests \( I'(\theta_H) \) in the high state and nothing otherwise, regardless of the level of available resources. Similarly, shareholders will leave the multidivision frm with at most \( I'(\theta_1) + I'(\theta_H) \) resources. In the high state, then, the multidivision frm invests \( I'(\theta_1) \) in unit 1 and \( I'(\theta_H) \) in unit 2, just like the separated frm. In the low state, the multidivision frm invests \( I'(\theta_1) \) in frm 1 when \( w \leq I'(\theta_1) \), and \( I'(\theta_1) \) otherwise, and nothing in frm 2. Hence, when \( w \leq I'(\theta_1) \), the multidivision and separated frms invest the same amounts, implying that \( \Delta = 0 \). When \( w > I'(\theta_1) \), however, the multidivision frm overinvests free cash ow in the low state, implying that \( \Delta < 0 \).

**Proof of Proposition 1.** The value function is continuous in \( I \) and the equilibrium investment level is a continuous function of \( w \). Therefore, the relation between \( \Delta \) and \( w \) is also continuous.

Note that \( \partial \Delta / \partial w = 0 \) in regions I and VII of Table I. In order to characterize the relation in the other five regions, we make use of the fact that

\[
\frac{\partial \Delta}{\partial w} = \frac{\partial V^M}{\partial w} - \frac{\partial V^S}{\partial w} + \alpha \left( \frac{\partial E[I^*]}{\partial w} - \frac{\partial E[I^{**}]}{\partial w} \right).
\]
where $E[I^J]$ is the expected value of investment by the multidivision ($J = \ast \ast$) and separated firms ($J = \ast$).

Consider the term $\partial V^M/\partial w$. In regions II and III, the multidivision firm uses external financing in the high state, so the marginal value of resources is $1 + C$. In the low state, it only uses internal funds and all resources are devoted to division 1, for a marginal value of $\theta_1 - 2w$. The expected marginal value is then $\partial V^M/\partial w = \frac{1}{2}(1 + C) + \frac{1}{2}(\theta_1 - 2w)$. In region IV, the firm uses external financing in the high state, but is unconstrained in the low state, for an expected marginal value of $\partial V^M/\partial w = 1 + C/2$. In regions V and VI, external financing is not used. In the high state, resources are invested in the two businesses according to $I^J_1$ and $I^J_2$, so the value of the firm is $(\theta_1 - I^J_1)I^J_1 + (\theta_H - I^J_2)I^J_2$. The fact that $\partial I^J_1/\partial w = \partial I^J_2/\partial w = 1/2$ leads to a marginal value in the high state of $\frac{1}{2}(\theta_1 + \theta_H - 2w)$. Because the firm is unconstrained in the low state, the expected marginal value in regions V and VI is $\partial V^M/\partial w = \frac{1}{4}(\theta_1 + \theta_H - 2w + 2)$. In region VII, $\partial V^M/\partial w = 1$; marginal resources are paid out to shareholders in period 0.

Consider next $\partial V^S/\partial w$. In region II, firm 1 uses internal financing and firm 2 uses external financing in the high state. Because firm 1 receives all of the liquid resources in this region, the marginal value of resources overall is equal to the marginal value of firm 1, that is, $\partial V^S/\partial w = \theta_1 - 2w$. As in region II, in regions III–V firm 1 uses internal financing and firm 2 raises funds externally. The difference is that in regions III–V, marginal resources are allocated to firm 2, offsetting its cost of external financing. The expected marginal value of resources is then $\partial V^S/\partial w = 1 + C/2$. In region VI, neither firm 1 nor firm 2 resorts to external financing. In the divestiture phase, resources are allocated to the units according to $w_1$ and $w_2$, giving an expected value of $(\theta_1 - w_1)w_1 + \frac{1}{2}(\theta_H - w_2)w_2 + \frac{1}{2}w_2$. Because $\partial w_1/\partial w = 1/3$ and $\partial w_2/\partial w = 2/3$, the marginal value of liquid resources is $\partial V^S/\partial w = \frac{1}{2}(\theta_1 + \theta_H - 2w + 1)$.

When substituted into the expression for $\partial \Delta/\partial w$ and restated in terms of $I'$ and $I''$, these results imply that

$$
\frac{\partial \Delta}{\partial w} = \begin{cases} 
w - I'(\theta_1) & \text{region II;} \\
I''(\theta_1) - w - \alpha & \text{region III;} \\
0 & \text{region IV;} \\
\frac{1}{2}(I'(\theta_1) + I'(\theta_H) - w) - \alpha & \text{region V;} \\
\frac{1}{6}(w - I''(\theta_1) - I'(\theta)) + \frac{1}{3}\alpha & \text{region VI.}
\end{cases}
$$

The nonmonotonicity and Fig. 3 follow immediately from the definitions of the regions. The fact that $\Delta = 0$ when $\alpha = 0$ and $w$ is sufficiently large can be seen by noting that investment levels in region VII do not depend on the form of organization when $\alpha = 0$. ■
Proof of Proposition 2. A change in $C$ has no effect on $\Delta$ when $w$ is in region I. We can use the envelope theorem in regions II–VII. From Eqs. (5) and (6), we have

$$\frac{\partial V^M}{\partial C} = -\frac{1}{2}(I_{1H}^{**} + I_{2H}^{**} - w)\lambda(I_{1H}^{**} + I_{2H}^{**} - w)$$

$$- \frac{1}{2}(I_{1L}^{**} - w)\lambda(I_{1L}^{**} - w) - \alpha \frac{\partial E[I^{**}]}{\partial C}.$$ 

The corresponding expression for $V^S$ comes from Eqs. (1) and (4):

$$\frac{\partial V^S}{\partial C} = -(I_1^* - w_1^*)\lambda(I_1^* - w_1^*) - \frac{1}{2}(I_{2H}^* - w_2^*)\lambda(I_{2H}^* - w_2^*) - \alpha \frac{\partial E[I^*]}{\partial C}.$$ 

These equations together with Lemmas 1–3 give

$$\frac{\partial \Delta}{\partial C} = \begin{cases} \frac{1}{2}(w - I'(\theta_1)) + \frac{1}{4}\alpha & \text{region II;} \\ \frac{1}{8}C & \text{regions III and IV;} \\ \frac{1}{8}(w^\circ + I'(\theta_H) - w) - \frac{1}{4}\alpha & \text{region V;} \\ 0 & \text{regions VI and VII;} \end{cases}$$

where $I'$ and $w^\circ$ are defined in Lemma 1. The derivative is positive in regions II–IV and zero in regions VI and VII. In region V it is negative when $w > w_1^* + I'(\theta_H) - w_1^*$. ■

Proof of Proposition 3. The envelope theorem gives

$$\frac{\partial \Delta}{\partial \theta_1} = \frac{1}{2}I_{1H}^{**} + \frac{1}{2}I_{1L}^{**} - I_1^* + \alpha \left( \frac{\partial E[I^*]}{\partial \theta_1} - \frac{\partial E[I^{**}]}{\partial \theta_1} \right).$$

This and Lemmas 2 and 3 imply that

$$\frac{\partial \Delta}{\partial \theta_1} = \begin{cases} \frac{1}{4}(I'(\theta_1) - w) - \frac{1}{4}\alpha & \text{region II;} \\ \frac{1}{4}(w - I''(\theta_1)) + \frac{1}{4}\alpha & \text{region III;} \\ 0 & \text{region IV;} \\ \frac{1}{4}(w - I'(\theta_1) - I'(\theta_H)) + \frac{1}{4}\alpha & \text{region V;} \\ \frac{1}{12}(I''(\theta_1) + I''(\theta_H) - \alpha - w) & \text{region VI;} \\ 0 & \text{region VII;} \end{cases}$$

where $I'$ and $I''$ are defined in Lemma 1. The derivative is negative in region II and positive in regions V and VI. The sign in region III depends on whether $w$ is greater than or less than $I''(\theta_1) + \alpha/2$. ■
Proof of Proposition 4. The envelope theorem implies that

\[
\frac{\partial \Delta}{\partial \theta_H} = \frac{1}{2}(I_{2H}^* - I_{2H}^{**}) + \alpha \left( \frac{\partial E[I^*]}{\partial \theta_H} - \frac{\partial E[I^{**}]}{\partial \theta_H} \right).
\]

This derivative is equal to zero in regions I–IV because investment in industry 2 is financed externally and is therefore equal regardless of organization. In region V, \( \frac{\partial \Delta}{\partial \theta_H} = \frac{1}{2}(\bar{I}_2 - I_1'(\theta_H)) + \alpha/4 > 0 \); and in region VI, \( \frac{\partial \Delta}{\partial \theta_H} = \frac{1}{12}(I''_1(\theta_1) + I''(\theta_H) - \alpha - w) \geq 0 \), which converges to zero as \( w \) approaches the upper boundary of the region.

Proof of Proposition 5. We want to show that strategic refocusing can be both feasible and optimal when \( I_1'(\theta_1 - Y) < I^d < I''(\theta_1 - Y) \) for some parameter values. As long as \( w \geq I^d \), deterrence can be achieved by allocating \( w_1 = I^d \) to the first division. Thus, deterrence is feasible for a sufficiently large \( w \).

To show optimality, consider the case where \( \Delta(\text{deters}) > 0 \), that is, where the firm would prefer to be diversified absent the entry threat, and suppose that deterrence is feasible, \( w \geq I^d \). As a notational convention, “\( \text{enters} \)” means the values are conditional on an industry 1 demand intercept of \( \theta_1 - Y \), and “\( \text{deters} \)” means conditional on an intercept of \( \theta_1 \). In this case, the only reason to refocus is if doing so deters entry. Refocusing is optimal, then, if the firm’s expected value as separate firms given \( w_1 = I^d \) and no entry, \( V^S(w_1 = I^d, \text{deters}) \), is greater than its expected value as a diversified firm given that entry takes place, \( V^M(\text{enters}) \). We can express this difference as

\[
\Phi = V^S(w_1 = I^d, \text{deters}) - V^M(\text{enters}) = \phi_1 + \phi_2 + \phi_3,
\]

where

\[
\begin{align*}
\phi_1 & \equiv V^M(\text{deters}) - V^M(\text{enters}); \\
\phi_2 & \equiv V^S(w_1 = I^d, \text{deters}) - V^S(w_1 = w_1^*, \text{deters}); \\
\phi_3 & \equiv V^S(w_1 = w_1^*, \text{deters}) - V^M(\text{deters}).
\end{align*}
\]

We want to show that \( \Phi > 0 \) for some parameter configurations.

First, choose \( C \) low enough so that \( |\phi_1| > |\phi_2| \). This is possible because \( \phi_1 \), the difference between the multidivision firm’s value without and with entry, is strictly positive, and \( \phi_3 = -\Delta \) approaches zero as \( C \) goes to zero. Then consider, \( \phi_2 \), the loss in value from refocusing with \( w_1 = I^d \) instead of \( w_1 = w_1^* \), where \( w_1^* \) is the optimal refocusing allocation in the problem without entry. Clearly, \( \phi_2 \) goes to zero as \( I^d \) approaches \( w_1^* \). We can choose a level of \( I^d \) arbitrarily close to \( w_1^* \) by adjusting \( Z \), and thereby make \( \phi_2 \) arbitrarily small. \( \blacksquare \)


