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Structural constraints on partisan bias under the efficient gerrymander *

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Abstract. Partisan bias occurs when the translation of the popular vote into legislative seats differs between competing parties. This paper contains a theoretical and empirical analysis of the consequences of an efficient gerrymander for the partisan bias of an electoral system. Under partisan apportionment, bias is shown to depend on some structural features of the electoral environment; namely, the size of the voting population and the number of single-member districts within a political jurisdiction. A statistical analysis reveals the predicted relationships in data on Congressional elections across states in the 1950–1984 period. This paper highlights the importance of some measurable features of the electoral environment for determining bias and provides an indirect test of partisan gerrymandering in congressional apportionment processes.

1. Introduction

Political outcomes in a representative democracy depend on the allocation of voters among legislative districts. This fact has fueled a large literature studying the characteristics of legislative apportionment. Partisan bias is one important consequence of legislative apportionment.[1] Partisan bias is a measure of the degree to which the translation of the popular vote into legislative seats differs across competing parties. In recent years, considerable progress has been made in the identification and estimation of partisan bias.[2] Progress has also been made in explaining variation in partisan bias across legislatures and within legislatures over time.[3]

Partisan bias is important for evaluations of alternative systems of democratic representation. Yet, while empirical developments in its estimation have been impressive, the theoretical determinants of partisan bias are largely

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unexplored. There are few analyses identifying the precise relationship between alternative apportionment processes and partisan bias. One major purpose of this paper is to make clear the theoretical relationship between a specific form of legislative apportionment – the efficient gerrymander – and partisan bias. Our second purpose is evaluate the predicted relationships using data on congressional elections from the 1950–1984 period.

There are at least three motivations for this study. First, while many of the implications of an efficient gerrymander have been exhaustively explored, no analyses of which we are aware relate this form of apportionment to the definition of partisan bias utilized in the contemporary empirical literature (Grofman, 1975; and Owen and Grofman, 1988). Thus, one contribution of this paper is to develop a theoretical model that generates predictions about the relationship between gerrymandering and partisan bias within the context of modern empirical analyses.

Second, partisan bias is often attributed to gerrymandering; the deliberate construction of legislative districts to enhance the election prospects of a party's members. Indeed, there is ample evidence of attempts to control the redistricting process for the purpose of partisan advantage (Abramowitz, 1983; Gopoian and West, 1984; Born, 1985; Cranor, Crawley, and Scheele, 1989; and Niemi and Winsky, 1992). But even though the potential consequences of gerrymandering for the partisan composition of a legislature are recognized, attempts to identify the existence and magnitude of partisan bias yield mixed results. Many scholars find little evidence of any strong or lasting consequence of partisan apportionment (Scarrow, 1982; Glazer, Grofman, and Robbins, 1987; and Campagna and Grofman, 1990). And some thoughtful students of apportionment suggest that the effects of the partisan gerrymander are either short-lived or exaggerated (Cain, 1985; and Niemi and Jackman, 1991). Our analysis provides a novel indirect test of the consequences of gerrymandering for partisan bias.

And third, legislative apportionment is sometimes the center of popular controversy and often the object of legal conflict. Since *Baker* v. *Carr* (369 U.S. 186, 1962), the so-called "one-man, one-vote" ruling, courts have vigorously attacked apportionment plans said to violate equal protection provisions of the constitution. The primary focus on this ruling is invariably racial vote dilution. However, courts have generally found partisan gerrymandering nonjusticiable. In *Davis* v. *Bandemer* (478 U.S. 109, 1986), the Supreme Court let stand an admittedly partisan gerrymander of Indiana by the GOP. The majority decision (signed by Justices White, Brennan, Marshall, and Blackmun) held that although the gerrymander had a "discriminatory intent", not enough evidence existed to demonstrate a "discriminatory effect". Our analysis illuminates several key factors for determining the consequences of partisan apportionment.

We begin with a theoretical analysis of the consequences of the efficient gerrymander – the allocation of voters by a political party to obtain maximum partisan advantage – for partisan bias. We show that under the efficient gerrymander, partisan bias depends on some key structural features of the representative process; namely, the size of the voting population and number of single-member districts in a political jurisdiction (e.g., state). We find that, all else equal, under the efficient gerrymander an increase in the size of the voting population within a political jurisdiction increases partisan bias while an increase in the number of single-member districts within a political jurisdiction decreases partisan bias. A proportionate increase in both voting population and the number of single-member districts in the jurisdiction (i.e., increases that retain the same voting population per district) decreases partisan bias. One implication of our analysis is that an electoral system that experiences an increase in the size of its voting population without a proportionate increase in the number of single-member districts into which that population is apportioned (e.g., Congress, over time) should exhibit more partisan bias. Thus, partisan bias across unrelated political jurisdictions (e.g., states) is systematically and directly related to the size of the voting population is each district within these jurisdictions.

We test these predictions on data from U.S. House elections and state congressional delegations over the period 1950–1994. Our theory suggests that, conditional on the likelihood of partisan apportionment, partisan bias should be increasing in the size of a state's population and decreasing in the size of its congressional delegation. We find, in fact, that the sensitivity of partisan bias to increases in relative party strength, a proxy for one-party control of a state's political institutions, grows with a state's population and diminishes with the size of a state's congressional delegation. Consistent with our theory, we find that conditional on the likelihood of partisan apportionment, partisan bias is affected by the size of a state's voting population and congressional delegation.

Many of our results are related to the extant literature. In the seminal work of Buchanan and Tullock (1962), they develop some basic relationships between the number of representatives per capita and the costs of collective decision-making. Specifically, a higher ratio of representatives to voters lowers these costs. Our analysis shows that the relationship between per capita representation and the external costs of collective decision-making is tempered by partisan gerrymandering and, more generally, shows why these costs depend on electoral systems. Thus, our results augments Buchanan and Tullock's analysis of the effects of varying degrees of representation.

Additionally, in a recent paper, Gelman and King (1994) point out that reapportionment is frequently bipartisan and, even when partisan, serves to reverse or moderate previous partisan gerrymanders. Examining the lower houses of state legislatures over the 1968–1988 period, they show that redistricting, on average, tends to reduce the degree of partisan bias in state legislative elections. Gelman and King's (1994) chief message is that general propositions about the effects of partisan apportionment for partisan bias must be tempered by knowledge of a state's political climate (e.g., party competition) and historical record (e.g., the partisan bias extant at redistricting). Like Gelman and King (1994), our results suggest that unqualified statements about the consequences of partisan apportionment for partisan bias are not supported by the data and that voting population and legislature size are important conditioning variables. Unlike Gelman and King (1994), however, our results suggest that the potential consequences of partisan apportionment are long-lived and related to the size and legislative structure of a state.

Others have recognized the importance of the size of a state's voting population and congressional delegation for understanding the relationship between the allocation of legislative seats and the popular vote. For example, Taagepera (1973) demonstrates the importance of these key variables for explaining responsiveness, the sensitivity of the distribution of legislative seats to changes in the vote distribution between parties. Taagepera's insight is gained by recognizing that when all voters are contained in one at-large district, responsiveness must be infinite at the critical vote share (e.g., 50%). Since there is only one contested seat, the proportion of seats held by any party is either zero or one hundred percent. On the other hand, absolute proportional responsiveness (e.g., a one-percent change in a party's vote share results in a one-percent change in its seat share) can only obtain when the size of the voting population and the number of contested seats are identical. King and Browning (1987) confirm Taagepera's insight in empirical estimates of the determinants of responsiveness in Congressional elections. Our results complement Taagepera's in confirming the importance of voting population and delegation size for understanding the mapping of votes into seats. Our results are somewhat different, though, in that we illustrate these variables are essential for understanding the determinants of partisan bias as well.

Understanding the determinants of partisan bias is important for evaluations of representative systems (Balinski and Young, 1982). Understanding these determinants may also prove useful for explaining variation in public decisions. In a companion paper, we show that over the 1960–1990 period states with larger legislatures have higher per capita expenditures even after controlling for party and demographic effects (Gilligan and Matsusaka, 1994). This effect is strongest for the upper chambers of state legislatures where each additional seat increases per capita spending at the margin by roughly \$10. While there are alternative theoretical explanations for these findings, they may suggest that bias, regardless the direction, is systematically related to such important public decisions as spending and taxation. Exploring this conjecture obviously requires further inquiry.

The next section of the paper contains the theoretical analysis used to derive our results. Section 3 presents our empirical analysis and findings. Section 4 is a brief conclusion.

2. Partisan bias under the efficient gerrymander

Partisan bias is one characteristic of any apportionment plan that maps the distribution of the popular vote into the distribution of elected legislators between competing parties. Partisan bias is a measure of the dependence of the distribution of elected legislators by political party on party identity. That is, partisan bias is present if a given vote share generates different seat shares for each party. A large and increasingly sophisticated empirical literature has attempted to measure this characteristic across national and state legislatures. In this section of the paper we attempt to complement this empirical literature by developing a theoretical foundation which illustrates the dependence of partisan bias on voting population size and the number of single-member districts within a political jurisdiction under an efficient gerrymander.

2.1. *Assumptions and the model*

Consider a political jurisdiction with a population of $i = 1, \ldots N$ odd voters and index variables $c_i^v \in \{0, 1\}$ that identify a voter's preference for a legislator in one of two competing parties. The number of voters favoring legislators from each competing party, labeled 0 and 1, are $N - \sum_{-1}^{N} c_i^v$ and $\sum_{i=1}^{N} c_i^v$, respectively. The median voter favors a party 1 legislator if and only if $\sum_{i=1}^{N} c_i^v \geq (N + 1)2$.

Voters elect representatives who formulate government policies in a legislature. Representatives are elected from single-member districts containing an equal number of voters. Let K be the number of districts in the legislature and N/K the number of voters in each district. For purposes of exposition, assume that N/K is an integer and, therefore, that K is odd.

A given district $k = 1, \ldots K$ is a collection of N/K voters and party vote shares. The median vote in district $k \in K$ is for a party 1 legislator if and only if $\sum_{i=1}^{N/K} c_i^v \geq ((N/K) + 1)/2, i \in k$. As suggested by Black's Median Voter Theorem (1958), we assume that the political affiliation of each elected legislator, denoted $c_k^l \in \{0, 1\}$, is the same as the median vote in the district. The median legislator belongs to party 1 if and only if $\sum_{k=1}^{K} c_k^l \geq (K+1)/2$.

2.2. *An illustrative sketch*

The model maps the distribution of the popular vote between competing parties into the distribution of elected legislators by party identity. The precise form of this mapping depends on the way in which voters are allocated across the K legislative districts which, of course, depends on the details of the utilized apportionment process. Thus, any apportionment plan is a function $A : \{c_1^v, \ldots, c_N^v\} \rightarrow \{c_1^l, \ldots, c_K^l\}$. Any measure of partisan bias is itself a function of the apportionment plan A.

To illustrate, consider an example in which $N = 15$, $K = 5$, $c_i^v = 0$ for $i \leq 7$ and $c_i^v = 1$ otherwise. Clearly, $\sum_{i=1}^{N} c_i^v \geq (N + 1)/2$ and the median vote is for party 1. Consider an allocation of voters based on their arbitrary index numbers. That is, the first three voters are assigned to the first district, the second three voters to the second district, and so forth. Then, in order, the five legislative districts contain the following party configuration of voters $\{(0, 0, 0), (0, 0, 0), (0, 1, 1), (1, 1, 1), (1, 1, 1)\}$, implying $c_k^l = 0$ for $k \leq 2$, $c_k^l = 1$ for $k > 2$, and $\sum_{i=1}^{K} c_k^l \geq (K + 1)/2$; the median legislator belongs to party 1. Given this allocation of voters, party 1's simple electoral majority maps into a simple legislative majority as well; casually speaking, this electoral system is relatively unbiased.

Consider an alternative allocation of voters that combines two type $c_i^v = 1$ voters with one type $c_i^v = 0$ voter for as many districts as possible. Then $c_k^l = 1$ for $k \leq 4$, $c_k^l = 0$ for $k = 5$, and $\sum_{i=1}^{K} c_k^l \geq (K + 1)/2$; the median legislator again belongs to party 1. Given this allocation of voters, party 1's simple electoral majority maps into a substantial legislative super-majority (i.e., 80%); this electoral system is arguably biased towards party 1.

And lastly, consider a complimentary allocation which combines two type $c_i^v = 0$ voters with one type $c_i^v = 1$ voter in as many districts as possible. Then $c_k^l = 0$ for $k \leq 3$, $c_k^l = 1$ for $k > 3$, and $\sum_{i=1}^{K} c_k^l < (K + 1)/2$; the median legislator is belongs to party 0. Given this allocation of voters, party 1's simple electoral majority maps into a simple legislative minority; this electoral system is arguably biased towards party 0.

2.3. *Definition of partisan bias*

The previous example and the extant literature suggest a natural definition of partisan bias within the framework of our model. This definition is based on the relative proportion of legislative seats gained by a party when the vote share between the two parties is roughly equal; that is, when $\sum_{i=1}^{N} c_i^v = (N+1)/2$. Specifically, we assume that partisan bias is given by the measure

$$\beta(A) = \ln\left[\frac{\sum_{k=1}^{K} c_k^l/K}{1-\sum_{k=1}^{K} c_k^l/K}\right]$$

whenever

$$\sum_{i=1}^{N} c_i^v = (N+1)/2.$$

The measure $\beta(A)$ has many desirable properties. First, for a sufficiently large legislature this measure approximates zero whenever party identity is irrelevant to the votes-to-seats mapping. There is no partisan bias when roughly equal vote shares (e.g., $\sum_{i=1}^{N} c_i^v = (N+1)/2$ are associated with roughly equal seat shares (e.g., $\sum_{k=1}^{K} c_k^l = (K+1)/2$ and, indeed, $\beta(A) \approx 0$. Second, this measure runs from negative infinity whenever the electoral systems is maximally biased in favor of party 0 to positive infinity when the system maximally favors the other party.[4] And third, this measure of bias is symmetric about zero. For example, whenever party 0 wins two-thirds of the seats on an equal vote share, $\beta(A) = -.69$. When party 1 wins two-thirds of the legislature upon enjoying half the vote, $\beta(A) = .69$. Thus, the absolute value of $\beta(A)$ is a good quantitative measure of the amount of bias present in an electoral system.

2.4. *The efficient gerrymander*

Consider the following apportionment process, denoted A_1, designed to maximize the number of legislators from party 1 regardless the popular vote. This is easily accomplished by combining (N+K)/2K party 1 voters with (N–K)/2K party 0 voters in as many districts as possible. The value (N+K)/2K is the smallest integer greater than N/2K, exactly half the voting population of a legislative district, and thus guarantees the smallest winning majority in each legislative district. This assignment process amplifies party 1 voters by making them simple majorities in as many districts as possible. This apportionment process illustrates partisan efficiency in that the party controlling apportionment (in this case, party 1) maximizes the seats it obtains for any vote total. This algorithm has many of the properties thought to characterize an efficient partisan gerrymander, including larger electoral victories for legislators from the party not controlling the apportionment process (Caine, 1984, 1985; Owen and Grofman, 1988; and Cranor, Crawley, and Scheele, 1989).

2.5. *Partisan bias under the efficient gerrymander*

Under the efficient gerrymander favoring party 1, A_1, the number of seats obtained by party 1, $\sum_{k=1}^{K} c_k^l$, must satisfy the constraint $\sum_{k=1}^{K} c_k^l (N+K)/2K \leq \sum_{i=1}^{N} c_i^v$; the allocation of party 1 voters across districts won by party 1 legislators (the quantity to the left of the inequality sign) must not exhaust its supply of voters (the quantity to the right of the inequality sign). Solving this constraint for party 1's seat proportion and substituting into our definition generates a measure of partisan bias under the efficient gerrymander.

$$\beta(N, K; A_1) \leq \ln[(N+1)/(K-1)]. \tag{2.1}$$

The right-hand side of Equation (2.1) identifies the maximum bias attainable by a gerrymandering party as a function of the political jurisdiction's voting population and number of single-member districts. Our first result then is evident.

Proposition 1. Partisan bias depends on the number of voters and single-member districts within a political jurisdiction.

One implication of Proposition 1 is that estimates of partisan bias (either across political jurisdictions or over time within a jurisdiction) should account for the elemental electoral parameters – voting population and legislature size – of the political jurisdiction.

Inspection of Equation (2.1) yields some additional results, as well.

Proposition 2. Partisan bias is non-decreasing in N and non-increasing in K.

Partisan bias is (weakly) greater for political jurisdictions with larger voting populations and fewer single-member districts. Recall that the strategy of the party attempting to construct the efficient partisan gerrymander is to utilize its voters most effectively while concentrating the opposition's voters into as few districts as possible. All else equal, the larger the voting population and the fewer the number of legislative districts, the more successful is the controlling party's strategy.

The intuition for these comparative statics results are fairly straightforward and are best understood by recalling the constraint that identifies the maximum number of seats attainable by a party employing the efficient partisan gerrymander, $\sum_{k=1}^{K} c_k^l (N+K)/2K \leq \sum_{i=1}^{N} c_i^v$. Again, this constraint says that the gerrymandering party must place a simple majority of voters in any district it wins. An increase in the voting population helps the gerrymandering party since its new voters (by definition, approximately half of the increase in voting population) are allocated efficiently to obtain legislative seats while the other party's new voters are concentrated ineffectively in safe districts. An increase in the number of legislative districts hurts the gerrymandering party since the opposition party has more surplus (i.e., ineffectively allocated) voters with which to contest any new seats. For sufficiently large legislatures, partisan bias goes to zero as the size of the voting population approaches the number of contested seats. Again, the intuition is straightforward. Partisan bias under the efficient gerrymander results from concentrating opposition voters into a strategically small number of districts. If such concentration is not possible because every voter has (is) a representative, no partisan bias results.

In some electoral systems, voting population size and the number of legislative districts are closely related across political jurisdictions. Letting $j = 1, \ldots, J$ index jurisdictions, each legislative district contains approximately the same and a constant number of voters; $c = N_j/K_j$. The House of Representatives of the United States Congress is one such system. Given the comparative statics reported in Proposition 2, it is not obvious whether larger states (e.g., states with larger voting populations and proportionately larger congressional delegations) in such an electoral system would exhibit a greater potential for partisan bias.

Under the constraint of equal proportionate representation across political jurisdictions, any increase in the number of legislative districts contained in a political jurisdiction is accompanied by a given increase in voting population. Specifically, an increase by one in the number of single-member districts must be accompanied by an increase of c in the political jurisdiction's voting population. Given such a system,

Proposition 3. Partisan bias is non-increasing when both N and K are increased proportionately.

When multiple political jurisdictions (e.g., states) with different size voting populations satisfy a common voting population per district constraint, bias is (weakly) larger for jurisdictions with smaller voting populations and fewer single-member districts.

2.6. *Implications of the analysis*

The results of the analysis illustrate that partisan bias is related to the structural features – voting population and legislature size – of the electoral environment under the efficient gerrymandering. Specifically, the analysis suggests that changes in voting population and legislature size affect partisan bias. A growing voting population, holding legislature size constant, increases partisan bias. An increase in the number of single-member districts within a jurisdiction, holding voting population constant, decreases partisan bias. And partisan bias is decreased by proportionate increases in voting population size and the number of single-member districts.

Two important implications are evident from the analysis. First, attempts to explain variation in partisan bias across jurisdictions should, to the extent that partisan gerrymandering is thought important, consider these important structural features. Attempts to estimate bias in partisan environments without incorporating the structural features of voting population and legislature size may lead to empirical misspecification and spurious or weak results. Second, the existence of partisan gerrymandering can be indirectly detected by documenting the predicted empirical relations identified above. We do this in the next section of the paper.

3. Empirical determinants of partisan bias

The theoretical results of the previous section suggest that, under the efficient gerrymander, partisan bias is related to the size of a legislature and the voting population. In this section of the paper we explore the empirical validity of this prediction.

3.1. *The measurement of partisan bias*

Any measure of partisan bias must be based on an estimate of the mapping between the distributions of the popular vote and legislative seats across political parties. The following empirical model

$$s_i = K_i\{1 + \exp[g(V_i; \beta_o, \beta_1)]\}^{-1} + \varepsilon_i \qquad (3.1)$$

provides such an estimate where s_i is the number of legislators from the Democratic party seated in the ith election, K_i the total number of single-ember legislative districts contested in the ith election, V_i the proportion of votes cast for Democratic candidates in the ith election, $g(V_i; \beta_0, \beta_1) = -\ln\beta_0 - \beta_1 \ln[V_i/(1 - V_i)]$ where $\ln\beta_0$ and β_1 are estimated parameters, respectively, and ε_i is a random disturbance term drawn from a binomial distribution such that $E(\varepsilon_i) = 0$.

Equation (3.1) possesses many advantages for estimating the mapping from votes to seats. Equation (3.1) distinguishes between the concepts of partisan bias, the asymmetric translation of votes into seats across competing parties, and responsiveness, the sensitivity of seat proportion to changes in vote shares.[5] The partisan bias parameter $\ln\beta_0$ identifies any asymmetric treatment of competing parties in the representative process. This parameter measures the share of the seat proportion total obtained by a party that is independent of that party's share of the popular vote. When $\ln\beta_0 = 0$, the Democratic seat proportion is fully explained by their share of the popular vote. When $\ln\beta_0 > 0$, the representative process is biased in favor of Democratic candidates while the opposite is true when $\ln\beta_0 < 0$.

Equation (3.1) also imposes some necessary definition restrictions. First, it is evident that in the absence of partisan bias, a party that receives all of the popular vote should be allocated all of the seats in a legislature. Thus, any functional form used to estimate the vote-seat relationship should pass through the zero-percent vote, zero-percent seat (0,0) and 100-percent vote, 100-percent seats (1,1) points. And since, by definition, bias is absent if and only if parties are treated symmetrically by the representative system, the vote–seat relationship under unbiased two-party competition must intercept the 50-percent vote, 50-percent seat point (1/2, 1/2). Equation (3.1) satisfies both of these restrictions.

King and Browning (1987) estimate Equation (3.1) using data from elections to the United States House of Representatives over the period 1950–1984. That is, King and Browning (1987) estimate the number of congressional seats assigned to Democratic candidates in a state as a function of the share of the vote received by all Democratic candidates in that state's House elections. The equation is estimated separately for each state (excluding Alaska and Hawaii, all at-large states, and Nebraska given its non-partisan apportionment process) over the entire time period. King and Browning (1987) support this particular pooling method by arguing that patterns of geographical and political diversity are more stable across time than states and that the apportionment of congressional districts is primarily a matter for the individual states. King and Browning (1987) also conduct several empirical tests which they report supports their pooling regime. A primary contribution of King and Browning (1987), then, is to provide a set of (normally distributed) maximum likelihood estimates of the partisan bias (and responsiveness) parameter based on congressional elections across states for the 1950–1984 period. Graciously, the authors provided these and most of the accompanying data used in our analysis.[6]

Some summary statistics of the partisan bias estimates are reported in Table 1 of this paper. As inspection indicates, the average of the estimated partisan bias coefficients is close to zero. Nearly 60% of the estimated coefficients of partisan bias are symmetrically distributed about zero and contained within $-.30 < \ln\beta_0 < .30$. That is, for most states the magnitude of partisan bias restricts a party receiving half the popular vote from obtaining more than 58% of contested seats. One interpretation of this finding is that partisan bias is of minor importance in many of the states for congressional elec-

Table 1. Summary statistics of variables

Variable	Mean	Standard error	Minimum	Maximum
Estimate of partisan bias ($\ln(\beta_0)$)	–.029	.427	–.920	1.120
Standard error of bias estimate	.849	.679	.100	3.600
Measure of ideology (unweighted)	.152	.083	.027	.396
Measure of ideology (weighted)	.151	.081	.007	.333
Measure of party strength	.624	.177	.342	.948
Average # of legislative seats	9.744	9.087	1.571	40.000
Average population (M)	4.128	3.885	.628	16.751

All of the variables except the size of a state's legislative delegation and its voting population were provided by King and Browning (1987). The size of a state's voting population and congressional delegation were obtained from the *Information Please Almanac* (1964, 1984). The numbers reported i the table and used in the analysis were weighted by the number of data years within a decade (i.e., data from the 1980s are weighted less).

tions during this time period. However, there are some states with substantial Republican (Kansas, Michigan and Ohio) and Democratic biases (Texas, California and Florida) over this period. Moreover, there is substantial variation in the partisan bias estimates across states.[7]

3.2. *Explaining partisan bias*

It is generally appreciated that partisan bias varies across political jurisdictions. Several authors have offered explanations for this variation. One source of this variation may be relative party strength. All things equal, it is easier for a political party to engage in partisan redistricting if that party dominates the political institutions within a state. The bias resulting from an attempt to maximize a party's control of a legislature through reapportionment should be greatest when that party faces little opposition from the competing political party. The party strength variable used in our analysis is the Ranney index which ranges from 0 for Republican dominated states to 1 for states whose political apparatus is controlled by the Democratic party (Ranney, 1976, 1985). This measure is based on the dominance of a party in securing state-level political offices and is calculated for the 1962–1980 period.[8] Since negative measures of partisan bias are associated with Republican advantage while positive measures favor the Democratic Party, a positive association between the King and Browning (1987) estimates of partisan bias and the Ranney index is expected.

Partisan bias is also thought to be greater in states with voting populations that are ideologically disposed toward one of the competing political parties. The measure of state-wide ideology (both weighted and unweighted) is based on polling data and ranges from –1 for states with the most liberal ideology to 1 for states with the most conservative ideology (Wright, Erikson, and McIver, 1985). A negative association between this ideology measure and the King and Browning (1987) estimates of partisan bias are expected.

King and Browning (1987) estimate the following equation

$$(\ln\beta_0)_j = \alpha_0 + \alpha_1 I_j + \alpha_2 PS_j + \mu_j \tag{3.2}$$

where $(\ln\beta_0)_j$ is their estimate of partisan bias for state j, I_j is the j state's measure of ideology, PS_j is Ranney's index of relative party strength for state, j, μ_j is the error term, and α_0, α_1, and α_2 are estimated parameters. As discussed above, the expectation is that $\alpha_1 < 0$ and $\alpha_2 > 0$. The frequency distributions of all variables used in these regressions are reported in Table 1. Table 1 also reports the sources of these data.

Equations (1) and (2) in Table 2 present the basic estimations of partisan bias (i.e., Equation (3.2)) and differ only in that equation (1) uses the unweighted while Equation (2) uses the weighted measures of ideology. To recall, the dependent variable in these weighted least-squares regressions is the King and Browning (1987) estimate of partisan bias in state congressional delegations during the 1950–1984 period. The weights in the regressions are the squared standard errors of the King and Browning (1987) partisan bias estimates. These weights are assumed proportional to the residual variances that obtain under ordinary least-squares regressions of partisan bias. The weighted least-squares regression method corrects for heteroscedastic disturbances that result under ordinary least-square regression of partisan bias measures that are estimated with varying degrees of precision.

As is evident from inspection of Equations (1) and (2) in Table 2, the regression results are consistent with the predictions. Both of the variables used to explain variation in partisan bias across the states have the expected sign. Moreover, the party strength measure is a statistically significant predictor of partisan bias (t-statistics of 2.77 and 2.95 in Equations (1) and (2), respectively). According to these regression results, a one-percent increase in Ranney's index of party strength (i.e., more control by the Democratic Party) correlates with slightly less than a one-percent (.83%) increase in the estimate of partisan bias (i.e., more partisan bias toward the Democrat Party). These results suggest that variation in partisan bias across states is related, as expected, to the lack of political competition within a state. The measure of state-level ideology is not significant for explaining variation in the bias estimates.

Table 2. Weighted least-square regressions of the King and Browning estimates of bias in congressional elections, 1950–1984[a]

	(1)	(2)	(3)	(4)
Constant	–.449*	–.454*	–.906*	–.882*
	(.201)	(.194)	(.397)	(.486)
Ideology	–.643	–.815	.519	.384
	(.594)	(.590)	(.701)	(.755)
Party strength	.895*	.931*	1.331*	1.310*
	(.323)	(.315)	(.669)	(.672)
Population	–	–	–3.939*	–3.855*
			(2.153)	(2.156)
Seats	–	–	1.704*	1.667*
			(.921)	(.923)
(Party strength)	–	–	7.573*	7.350*
× (population)			(3.950)	(3.943)
(Party strength) × (seats)	–	–	–3.273*	–3.175*
			(1.707)	(1.704)
F-test: Party strength	–	–	3.68*	3.47*
# Obs.	44	44	44	44
$\overline{R}^2$	.148	.148	.245	.239
SSE	.639	.639	.602	.604

[a]The weights in the regressions are the squared standard errors of the King and Browning (1987) estimates of partisan bias. Coefficients identified with an asterisk are significant at better than the five-percent level in one-tailed t-tests. The fourth to the last row report an F(1,37) which tests the null hypothesis that party strength is irrelevant for explanations of bias. Values of the test statistics with an asterisk are significant at better than the ten-percent level.

3.3. *Predictions of the theory*

The theoretical results developed above suggest that the size of a state's voting population and its congressional delegation should help explain variation in King and Browning's (1987) estimates of partisan bias.[9] Since our data are for elections to the U.S. House of Representatives and, by design, each district should contain roughly the same population, some hesitation is warranted about including both population size and the number of seats in a states congressional delegation in common regressions. That is, a state's population and congressional delegation size may be two highly correlated to offer independent explanatory power of partisan bias. Indeed, the simple correlation between a states' population and the size of its congressional delegation is greater than .99 in our data. Table 1 also reports the summary statistics for the state size data used in our analysis.

As might be expected given the pooling of data for the 1950–1984 period, however, the population per district for our sample is in fact not constant but rather increasing in the size of a state measured either by population or the size of its congressional delegation. This result is consistent with the fact that population growth rates are, on average, greater in larger states during this time period and apportionment reflects these growth differences only every ten years. Simple regressions of the variable population per district on a constant term and either voting population or the size of state's delegation reveals a statistically meaningful positive relationship. According to these simple regressions, a one standard deviation increase in the number of representatives in a state's delegation increases the state's predicted population per district by over 6,000 people (t-statistic 1.48). A one standard deviation increase in a state's voting population increases the state's voting population per district by slightly less than 6,500 people (t-statistic 1.60). These magnitudes constitute roughly a one-and-a-half percent increase in the population per district. As such, the larger states in our sample can be thought of as experiencing solitary increases in voting population (N) or decreases in (K). We exploit this variation to test the predictions of our model.

The theoretical results developed above suggest that, given partisan apportionment, the extent of bias in these data is related to the elemental parameters of a states electoral environment. Our theory implies that if the conditions for partisan bias are present – one-party control or substantial relative party strength – bias ought to be greater the larger a state's population and smaller its congressional delegation. Figure 1, which depicts the relationship between the measure of party strength and partisan bias found in regressions (1) and (2), illustrates these predicted effects. The flatter of the two lines drawn in Figure 1 depicts the relationship between party strength and partisan bias for a state of average size (both voting population and congressional delegation). The steeper line illustrates this same relationship for a state with a larger than average voting population or smaller than average congressional delegation. States with larger voting population and smaller congressional delegations are more susceptible to partisan bias as party competition subsides (i.e., as Ranney's party strength measure approaches zero and one).

The presence of these predicted effects can be detected with a simple modification of equation (3.2). As our discussion of Figure 1 suggests, the effect of a larger voting population and smaller congressional delegation is to steepen the regression line illustrating the relationship between relative party strength and partisan bias. This steepening, however, must still imply that

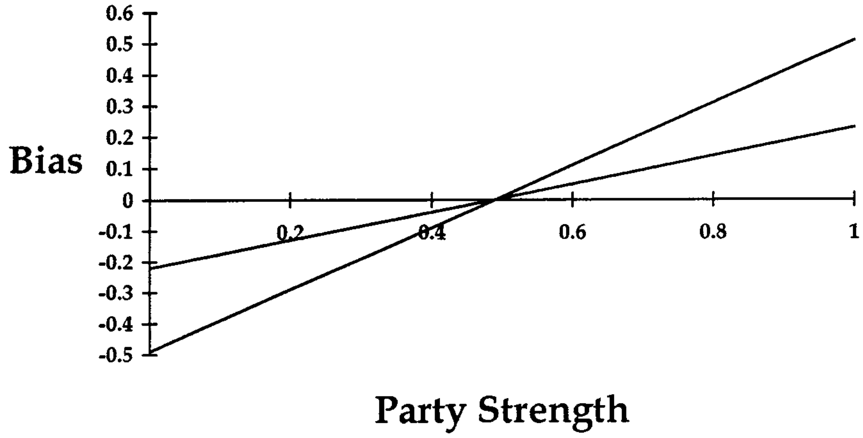

Figure 1.

partisan bias is unlikely to emerge (e.g., $(\ln\beta_0)_j \approx 0$) whenever the strength of the competing parties is relatively equal (e.g., $PS_j \approx .5$). This suggests that both the intercept term and slope coefficient on relative party strength should be expressed as functions of the size of a state's voting population, VP_j, and congressional delegation, CD_j. Given $\alpha_0 = \gamma_0 + \gamma_3 VP_j + \gamma_4 CD_j$ and $\alpha_2 = \gamma_2 + \gamma_5 VP_j + \gamma_6 CD_j$, we estimate the following equation

$$(\ln\beta_0)_j = \gamma_0+\gamma_1 I_j+\gamma_2 PS_j+\gamma_3 VP_j+\gamma_4 CD_j+\gamma_5 PS_j VP_j+\gamma_6 PS_j CD_j+\varepsilon_j \quad (3.3)$$

where, again, ε is a heteroscedastic error term reflecting differences in the precision with which partisan bias is estimates across states. Our theory predicts that the size of the population should steepen the relationship between party strength and partisan bias and, therefore, that $\gamma_3 < 0$ and $\gamma_5 > 0$. Since larger congressional delegations should flatten the relationship between relative party strength and partisan bias, $\gamma_4 > 0$ and $\gamma_6 < 0$. As before we expect $\gamma_1 < 0$ and $\gamma_2 > 0$.

3.4. *Estimating the structural constraint on partisan bias*

Equation (3) and (4) in Table 2 are estimates of Equation (3.3) and once again differ only with regard to the definition of the ideology variable (i.e., weighted *versus* un-weighted). Inspection of these equations illustrates several facts. First, it is clear that adding to Equation (3.2) the size of a state's population and congressional delegation and the interactions of these variables with Ranney's relative party strength measure increases one's ability to explain variation in partisan bias across the states. Relative to equations (1) and (2), the sum-of-squared error is lower for Equations (3) and (4). The adjusted r-squared is over 60% larger in the later two equations, as well. Moreover, an F(4,37-test)-test of the hypothesis that these additional variables are irrelevant for explanations of partisan bias can be rejected with a high degree of statistical significance.[10] The addition of the variables suggested by the theory developed above are important for helping to explain partisan bias.

Second, as before, Ranney's measure of relative party strength is an important predictor of partisan bias. An F(1,37)-test of the null hypothesis that party strength is not relevant for explanations of partisan bias, reported in Table 2, can be rejected at conventional levels of significance. Also similar to the results obtained from Equations (1) and (2), a one-percent increase in Ranney's measure of partisan strength correlates with a slightly less than one-percent (.69%) rise in the King and Browning (1987) estimate of partisan bias. The lack of party competition (i.e., high or low measures of relative party strength) has an observable effect on estimates of partisan bias in the states.

Third, all of the coefficients on the variables unique to Equation (3.3) are statistically significant at conventional levels and have the predicted sign. The size of a state's population steepens the relationship between relative party strength and partisan bias. The greater is a state's population the larger is the bias in Republican and Democrat controlled states. Referring to Figure 1, states with larger populations have lower (i.e., more negative, $\gamma_3 < 0$) intercepts and greater sensitivities (i.e., more positive, $\gamma_5 > 0$) of partisan bias to relative party strength. The opposite is true for states with larger congressional delegations. States with larger congressional delegations have higher (i.e., more positive, $\gamma_4 > 0$) intercepts and smaller sensitivities (i.e., more negative, $\gamma_6 < 0$) of partisan bias to relative party strength. The predictions of the theory developed above are evident in estimates of Equation (3.3).

A quantitative exercise illustrates the effects the size of a states' voting population and congressional delegation have on the relationship between relative party strength and partisan bias. Utilizing the results from Equation (3) in Table 2, one can derive the sensitivity of partisan bias to a small change in relative party strength (i.e., $\partial \ln\beta_0/\partial \text{PS} = .69$). This sensitivity is positive. One can also derive the fact that a one-percent increase in the size of a state's population increases this sensitivity by over 40% while a one-percent rise in the size of state's congressional delegation reduces this sensitivity by nearly 45%. These results suggest that the relationship between partisan bias and party competition is quite sensitive to changes in the elemental parameters of a state's electoral environment.

3.5. *Summary*

By way of summary it seems appropriate to acknowledge the nature of the results in support of our basic hypotheses. All of the relevant estimated parameters are highly significant. Moreover, and most importantly, it would be difficult to conceive of a statistical test of our chief hypotheses that more favorable to the null hypothesis of no effect. Recall that one of the assumptions of the King and Browning (1987) analysis is that their estimate of partisan bias for each state is constant throughout the entire 1950–1984 period. Recent results by Gelman and King (1994) suggest that some caution should be used in carrying this assumption too far. The same is true of the independent variables in our regression estimation which are assumed constant throughout the period and even constructed during different sub-periods within the sample period. Given the sources and magnitude of noise that is introduced in our empirical design by these difficulties, we view the evidence as at least suggesting the value of further inquiry on the effects of state size – voting population and congressional delegation – on the relationship between partisan apportionment and electoral bias.

4. Conclusions

This paper contains a theoretical model which illustrates the relationship between partisan bias across congressional districts as a function of the partisan gerrymandering of legislative elections. This relationship is shown to depend on some key structural variables of the states electoral environment; the size of a state’s voting population and the number of representatives in its congressional delegation. A statistical analysis revealed the predicted relationships between these variables and the King and Browning (1987) estimates of partisan bias in Congressional elections across states in the 1950–1984 period. This paper, thus, highlights some important structural constraints on partisan bias. As such, this paper provides some indirect evidence of the importance of partisan gerrymandering in congressional elections and contributes to our evolving understanding of the relationship between partisan apportionment and the characteristics of electoral systems.

Notes

1. Another important characteristic is responsiveness, a measure of the sensitivity of the distribution of legislative seats to changes in the vote distribution between parties.
2. For example, the consequences of legislative redistricting for partisan bias in state legislatures have been recently documented (Gelman and King, 1994).

3. For example, one obvious determinant of partisan bias is relative party strength. Jurisdictions with little competition between parties exhibit more partisan bias since the dominant party is free to translate its political support (i.e., votes) into substantial legislative control (i.e., seats) (Ranney, 1976). Empirically, these factors have been shown to be useful for explaining variation in bias across state congressional delegations in the post-war period (King and Browning, 1987). Gelman and King (1994) examine the temporal variation in bias across state legislatures.
4. Since $\lim_{x\to 0}\ln(\frac{x}{1-x}) = -\infty$ and $\lim_{x\to 1}\ln(\frac{x}{1-x}) = \infty$.
5. The responsiveness parameter, β_1, identifies various characteristics of the democratic process independent of bias. When $0 \leq \beta_1 < 1$, responsiveness is said to be anti-majoritarian in that changes in the composition of a legislature are relatively unresponsive to changes in vote shares across parties. When $\beta_1 = 1$, proportional responsiveness obtains since the change in vote share equals the change in seat shares. Values of $1 < \beta_1 < \infty$ are indicative of majoritarian responsiveness since vote share swings are related to disproportionately large changes in seat shares. And when $\beta_1 = \infty$, winner-take-all responsiveness obtains since minute changes in vote shares lead to a total change in the party of all seated delegates.
6. A particular caution is warranted in interpreting the results from use of this pooled data set. The early part of the data are generated in an environment absent *Baker* v. *Carr* (369 U.S. 186, 1962), the so-called "one-man, one-vote" Supreme Court ruling that established equal protection screens for all apportionment plans. It is how this ruling constrained partisan apportionment plans and, more to the point, how the ruling affected the relative relationship between seats and population, on the one hand, and partisan bias, on the other, **across** the states.
7. As King and Browning (1987) report, the distribution of the responsiveness parameter is trimodal with modes at one (approximately proportional), six (strongly majoritarian), and ten (essentially winner-take-all). Unlike previous analyses that constrained the bias parameter to zero, King and Browning (1987) find measures of responsiveness that are widely distributed and considerably different than that suggested by the venerable "cube law."
8. Ranney's index is the simple average of four variables; 1) the average percentage of the popular vote won by Democratic gubernatorial candidates, 2) the average percentage of the seats in a state's upper chamber won by Democrats, 3) the average percentage of the seats in the a state's lower chamber won by Democrats, and 4) the percentage of all terms of governor, lower and upper house in which the Democrats had control.
9. We use a state's total population as a proxy for its voting population.
10. The value of the test statistic is 2.33, which is significant at better than the ten-percent level.

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